Genomics, Networks, and Computational Concepts for Polytopic SUSY Representation Theory

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2021 DEC 03

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OUTLINE

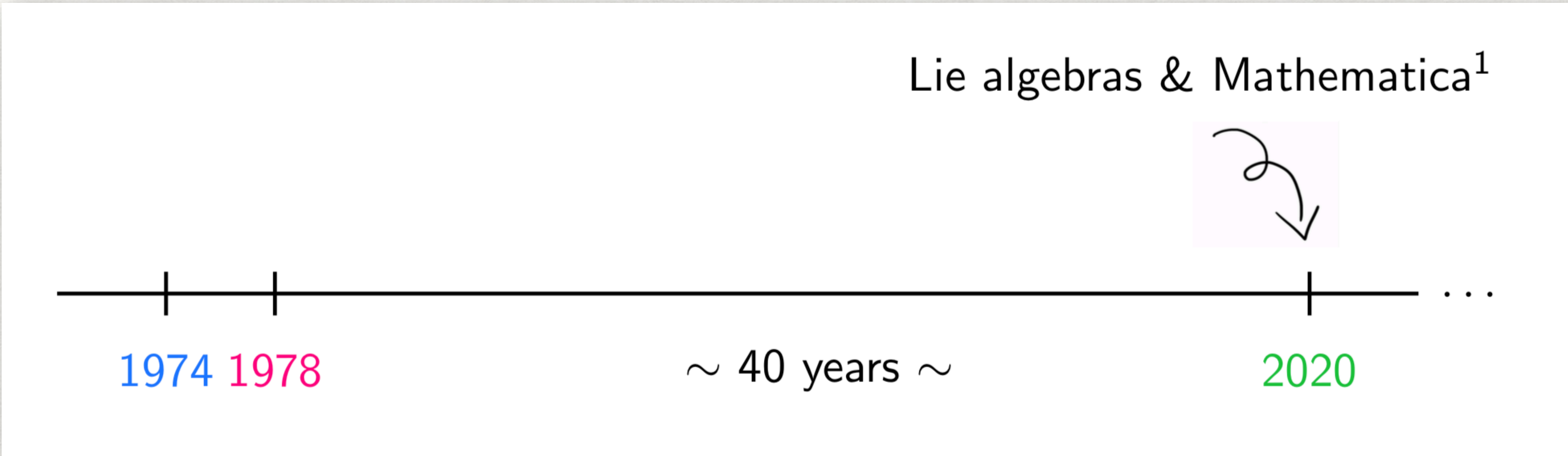
- A forty-year-old Problem
- The Ectoplasmic Conjecture
- Aynkrafields, Adynkra Digital Analysis (ADA) Scans, and 11D Supergravity Surprise
- SUSY Holography Conjecture
- Conclusions & Outlook

A 40-YEAR-OLD PROBLEM

"Our knowledge can only be finite, while our ignorance must necessarily be infinite."

— Karl Popper

A 40-YEAR-OLD PROBLEM: 11D SUPERFIELDS



- 1974: first 4D superfield written down [Salam, Strathdee, 1974]
- 1978: first 11D on-shell supergravity [Cremmer, Julia, Scherk, 1978]
- Irreducible off-shell formulation for the ten and eleven dimensional supergravity multiplet? Reducible one?
- 2020: 11D Superfield!! [Gates, YH, Mak, JHEP 09 089(2020)]

1: LieART [Feger, Kephart, 2012], SUSYno [Fonseca, 2011]

LINEARIZED NORDSTRÖM SUGRA

- [Gates, YH, Jiang, Mak, JHEP 1907 (2019) 063]: In Nordström theory, only non-conformal spin-0 part of graviton and non-conformal spin-1/2 part of gravitino show up

$$
\begin{aligned}
\mathrm{E}_\alpha &= \mathrm{D}_\alpha + \frac{1}{2}\Psi \mathrm{D}_\alpha \quad , \\
\mathrm{E}_{\underline{a}} &= \partial_{\underline{a}} + \Psi \partial_{\underline{a}} - i\frac{2}{5}(\sigma_{\underline{a}})^{\alpha\beta}(\mathrm{D}_\alpha \Psi)\mathrm{D}_\beta \quad .
\end{aligned}
$$

$$
\mathrm{E}_{\underline{a}}\Big| = \Big[\, 1 \; + \; \Psi \,\Big]\Big| \delta_{\underline{a}}{}^{\underline{m}} \partial_{\underline{m}} \; + \; \Big[- i\frac{2}{5}(\sigma_{\underline{a}})^{\alpha\beta}(\mathrm{D}_\alpha \Psi)\Big]\Big| \, \mathrm{D}_\beta
$$

$$
\mathrm{E}_{\underline{a}}\Big| = \mathrm{e}_{\underline{a}}{}^{\underline{m}} \partial_{\underline{m}} \; + \; \tilde{\psi}_{\underline{a}}{}^{\beta} \, \mathrm{D}_\beta
$$

$$
\mathrm{e}_{\underline{a}}{}^{\underline{m}} \; = \; \Big[\, 1 \; + \; \Psi \,\Big]\Big| \delta_{\underline{a}}{}^{\underline{m}} \quad , \quad \tilde{\psi}_{\underline{a}}{}^{\beta} \; = \; \Big[- i\frac{2}{5}(\sigma_{\underline{a}})^{\alpha\beta}(\mathrm{D}_\alpha \Psi)\Big]\Big|
$$

LINEARIZED NORDSTRÖM SUGRA

- All component fields of Nordström SG are obtained from spin-0 graviton, spin-1/2 gravitino, and all possible spinorial derivatives to the field strength $G_{\alpha\beta}$

$$T_{\alpha\underline{b}}{}^{\gamma} \;=\; -\;\frac{3}{10}\delta_{\alpha}{}^{\gamma}(\partial_{\underline{b}}\Psi) \;+\; \frac{3}{10}(\sigma_{\underline{b}}{}^{\underline{c}})_{\alpha}{}^{\gamma}(\partial_{\underline{c}}\Psi) \;+\; i\frac{1}{160}\left[-(\sigma^{[2]})_{\alpha}{}^{\gamma}(\sigma_{\underline{b}[2]})^{\beta\delta} \;+\; \frac{1}{3}(\sigma_{\underline{b}[3]})_{\alpha}{}^{\gamma}(\sigma^{[3]})^{\beta\delta}\right] G_{\beta\delta}$$

$$R_{\alpha\beta}{}^{\underline{de}} \;=\; -\; i\frac{6}{5}(\sigma^{[\underline{d}})_{\alpha\beta}(\partial^{\underline{e}]}\Psi) \;-\; \frac{1}{80}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta} \;+\; (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}}{}^{\underline{de}})^{\gamma\delta}\right] G_{\gamma\delta}$$

$$G_{\alpha\beta} = ([\mathrm{D}_{\alpha}, \mathrm{D}_{\beta}]\Psi)$$

SUPERSPACE

- D spacetime dimensional superspace: $(x^{\underline{a}}, \theta^{\alpha})$, where $\underline{a} = 0,1,2,\ldots,\mathrm{D}-1$ and $\alpha = 1,2,\ldots,d$. d is the number of real components of the spinors.

- Number of independent components in unconstrained scalar superfields is 2^d, where $n_B = n_F = 2^{d-1}$

Spacetime Dimension	Lorentz Group	Type of Spinors	d
11	SO(1,10)	Majorana	32
10	SO(1,9)	Majorana-Weyl	16
9	SO(1,8)	Pseudo-Majorana	16
8	SO(1,7)	Pseudo-Majorana	16
7	SO(1,6)	SU(2)-Majorana	16
6	SO(1,5)	SU(2)-Majorana-Weyl	8
5	SO(1,4)	SU(2)-Majorana	8
4	SO(1,3)	Majorana/Weyl	4

4D, $\mathcal{N} = 1$ SCALAR SUPERFILED

- General $\theta-$expansion of a scalar superfield in 4D, $\mathcal{N} = 1$ superspace

$$\begin{aligned}\mathcal{V}(x^{\underline{a}}, \theta^\alpha) \ &= v^{(0)}(x^{\underline{a}}) \ + \ \theta^\alpha \, v^{(1)}_\alpha(x^{\underline{a}}) \ + \ \theta^\alpha\theta^\beta \, v^{(2)}_{\alpha\beta}(x^{\underline{a}}) \\ &\quad + \ \theta^\alpha\theta^\beta\theta^\gamma \, v^{(3)}_{\alpha\beta\gamma}(x^{\underline{a}}) \ + \ \theta^\alpha\theta^\beta\theta^\gamma\theta^\delta \, v^{(4)}_{\alpha\beta\gamma\delta}(x^{\underline{a}})\end{aligned}$$

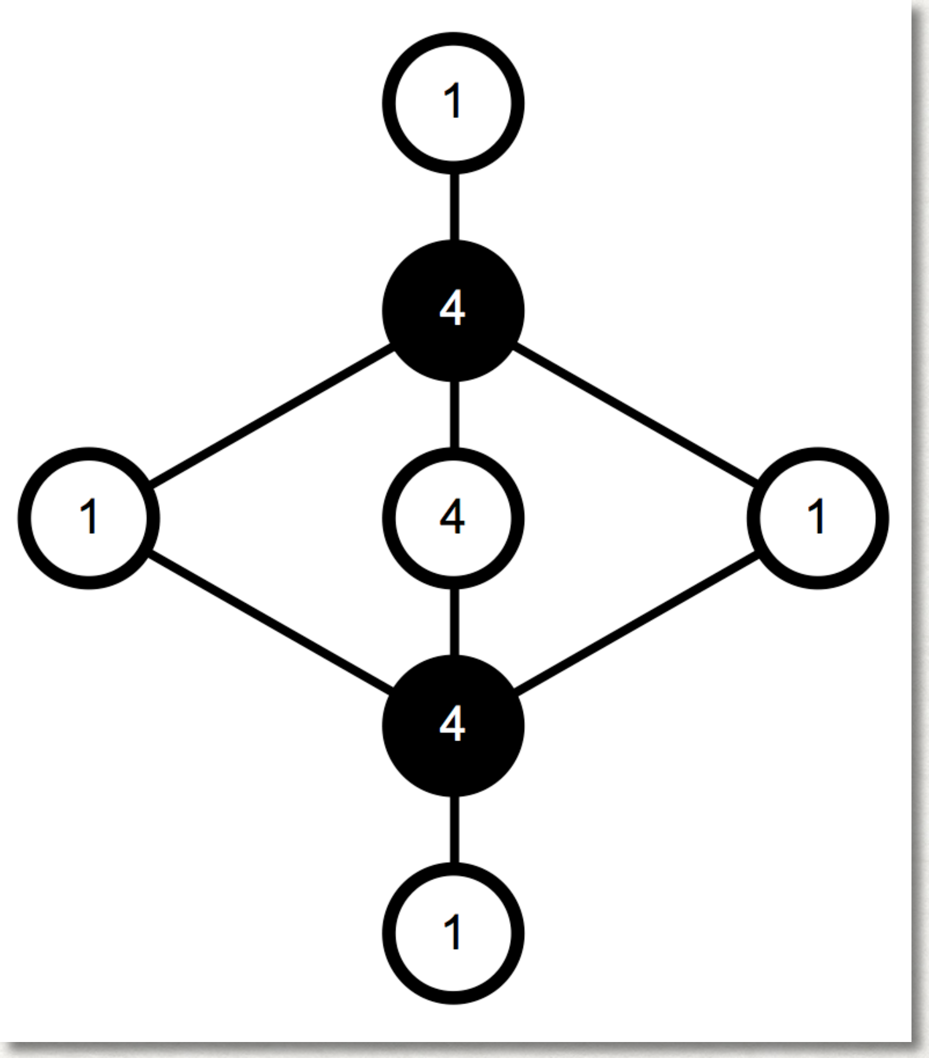
- Construct Irreducible $\theta-$monimials

- Level-0: no θ
- Level-1: $\{4\} \ = \ \theta^\alpha$
- Level-2: $\{1\} \ = \ \theta^\alpha\theta^\beta C_{\alpha\beta}$, $\{4\} \ = \ \theta^\alpha\theta^\beta i(\gamma^5\gamma^{\underline{a}})_{\alpha\beta}$, $\{1\} \ = \ \theta^\alpha\theta^\beta i(\gamma^5)_{\alpha\beta}$
- Level-3: $\{4\} \ = \ \theta^\alpha\theta^\beta\theta^\gamma C_{\alpha\beta}C_{\gamma\delta}$
- Level-4: $\{1\} \ = \ \theta^\alpha\theta^\beta\theta^\gamma\theta^\delta C_{\alpha\beta}C_{\gamma\delta}$

$$\boxed{\begin{aligned}\mathcal{V}(x^{\underline{a}}, \theta^\alpha) \ &= v^{(0)}(x^{\underline{a}}) \ + \ \theta^\alpha \, v^{(1)}_\alpha(x^{\underline{a}}) \ + \ \theta^\alpha\theta^\beta \Big[\, C_{\alpha\beta} \, v^{(2)}_1(x^{\underline{a}}) \ + \ i(\gamma^5)_{\alpha\beta} \, v^{(2)}_2(x^{\underline{a}}) \ + \ i(\gamma^5\gamma^{\underline{b}})_{\alpha\beta} \, v^{(2)}_{\underline{b}}(x^{\underline{a}}) \, \Big] \\ &\quad + \ \theta^\alpha\theta^\beta\theta^\gamma C_{\alpha\beta}C_{\gamma\delta} \, v^{(3)\delta}(x^{\underline{a}}) \ + \ \theta^\alpha\theta^\beta\theta^\gamma\theta^\delta C_{\alpha\beta}C_{\gamma\delta} \, v^{(4)}(x^{\underline{a}}) \quad .\end{aligned}}$$

4D, $\mathcal{N} = 1$ ADINKRA

Level	Component fields	Irrep(s) in $\mathfrak{so}(4)$
0	$f(x^{\underline{a}})$	$\{1\}$
1	$\psi_\alpha(x^{\underline{a}})$	$\{4\}$
2	$g(x^{\underline{a}}),\ h(x^{\underline{a}}),\ v_{\underline{b}}(x^{\underline{a}})$	$\{1\},\ \{1\},\ \{4\}$
3	$\chi^\delta(x^{\underline{a}})$	$\{4\}$
4	$N(x^{\underline{a}})$	$\{1\}$

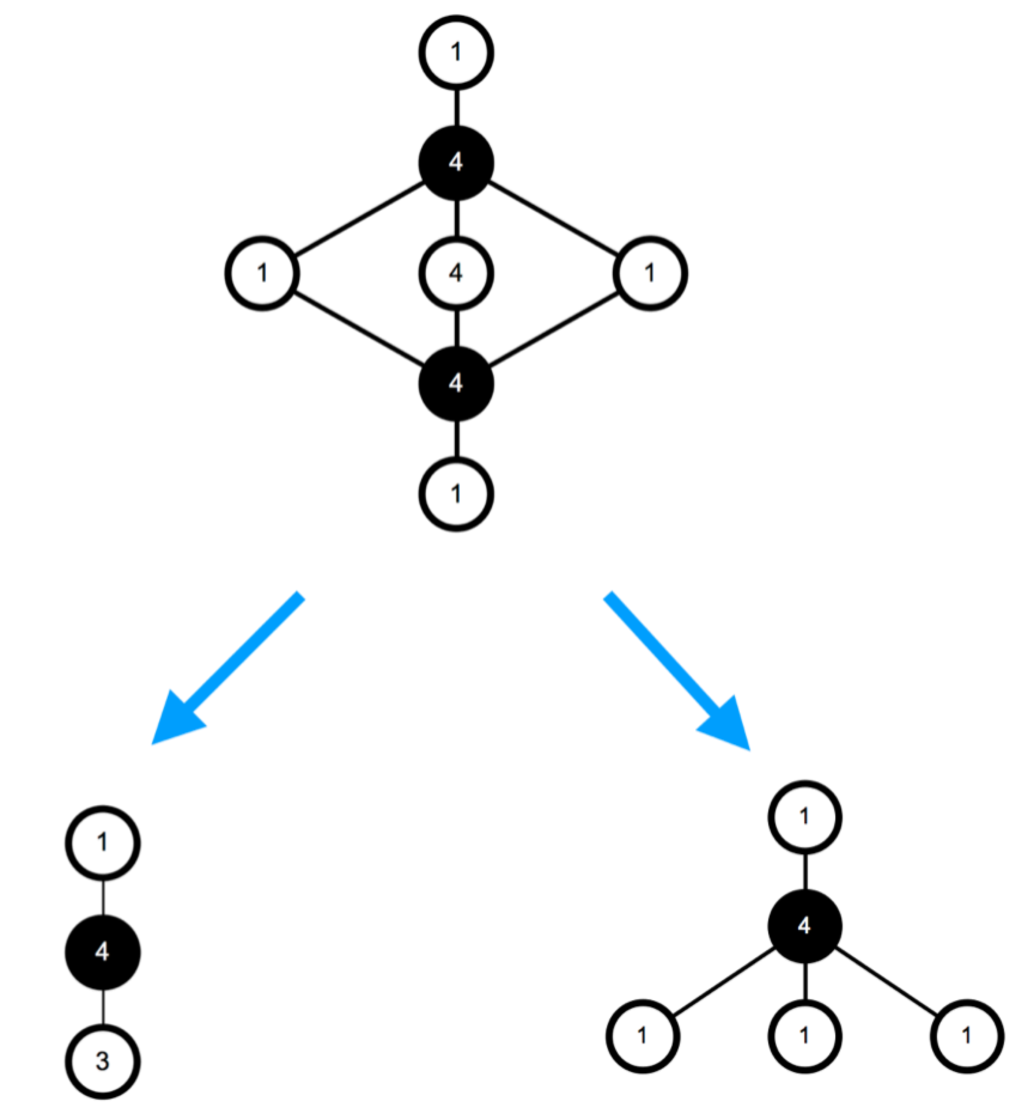


[Faux, Gates, 2005], [Doran, Faux, Gates, Hubsch, Iga, Landweber, 2008]

"The use of symbols to connote ideas which defy simple verbalization is perhaps one of the oldest of human traditions. "

CHIRAL & VECTOR SUPERMULTIPLETS

- Consider gauge conditions & chiral condition
- How to carry out the process for a general representation of spacetime supersymmetry is unknown! (Motivation for the adinkra approach to the study of superfields)



Vector Supermultiplet

Hodge-dual variants of the chiral supermultiplet

THE 4,294,967,296 PROBLEM

- In 11D, we have 32 Grassmann coordinates

$$\mathcal{V}(x,\theta) \;=\; v^{(0)}(x) + \sum_{n=1}^{32} v^{(n)}_{\alpha_1\cdots\alpha_n}(x)\,\theta^{\alpha_1}\cdots\theta^{\alpha_n}$$

- $2^{32} = 4{,}294{,}967{,}296$ total degrees of freedom

- Question: what irreducible representations of $\mathfrak{so}(1,10)$ occur among the 4,294,967,296 degrees of freedom in the scalar superfield?

- Until 2020, the answer was an unresolved puzzle.

TRADITIONAL PATH → THE 4,294,967,296 PROBLEM

- Start from constructing irreducible $\theta-$monomials
- Quadratic:

$$\begin{array}{ll} \{1\} & C_{\alpha\beta}\,\theta^\alpha\theta^\beta \\ \{165\} & (\gamma^{\underline{abc}})_{\alpha\beta}\,\theta^\alpha\theta^\beta \\ \{330\} & (\gamma^{\underline{abcd}})_{\alpha\beta}\,\theta^\alpha\theta^\beta \end{array}$$

- Cubic level: $[\ ]_{IR}$ means that a single γ-trace of the expression is by definition equal to zero.

$$\begin{array}{rlll} \{5,280\} & \left[(\gamma^{\underline{abcd}})_{\alpha\beta}\,\theta^\alpha\theta^\beta\,\theta^\gamma\right]_{IR} & & \\ \{3,520\} & \left[(\gamma^{\underline{abcd}})_{\alpha\beta}\,\theta^\alpha\theta^\beta\,(\gamma_{\underline{d}})_{\gamma\delta}\,\theta^\delta\right]_{IR} & , \quad \left[(\gamma^{\underline{abc}})_{\alpha\beta}\,\theta^\alpha\theta^\beta\,\theta^\gamma\right]_{IR} & \\ \{1,408\} & \left[(\gamma^{\underline{abcd}})_{\alpha\beta}\,\theta^\alpha\theta^\beta\,(\gamma_{\underline{cd}})_{\gamma\delta}\,\theta^\delta\right]_{IR} & , \quad \left[(\gamma^{\underline{abc}})_{\alpha\beta}\,\theta^\alpha\theta^\beta\,(\gamma_{\underline{c}})_{\gamma\delta}\,\theta^\delta\right]_{IR} & \\ \{320\} & \left[(\gamma^{\underline{abcd}})_{\alpha\beta}\,\theta^\alpha\theta^\beta\,(\gamma_{\underline{bcd}})_{\gamma\delta}\,\theta^\delta\right]_{IR} & , \quad \left[(\gamma^{\underline{abc}})_{\alpha\beta}\,\theta^\alpha\theta^\beta\,(\gamma_{\underline{bc}})_{\gamma\delta}\,\theta^\delta\right]_{IR} & \\ \{32\} & (\gamma^{\underline{abcd}})_{\alpha\beta}\,\theta^\alpha\theta^\beta\,(\gamma_{\underline{abcd}})_{\gamma\delta}\,\theta^\delta & , \quad (\gamma^{\underline{abc}})_{\alpha\beta}\,\theta^\alpha\theta^\beta\,(\gamma_{\underline{abc}})_{\gamma\delta}\,\theta^\delta & , \quad C_{\alpha\beta}\,\theta^\alpha\theta^\beta\,\theta^\gamma \end{array}$$

$$\frac{32\times 31\times 30}{3\,!} = \{4{,}960\} = \{32\}\oplus\{1{,}408\}\oplus\{3{,}520\}$$

THE FIRST SIGN OF TROUBLE

- $\theta-$monomials have multiple expressions

- You wouldn't know two versions of $\{320\}$ and $\{5{,}280\}$ are identically zero

    - Explicit proof by using Fierz identities presented in Appendix D of [Gates, YH, Mak, JHEP 09 089(2020)]

- Even for gamma matrix multiplications, you can get multiple expressions. e.g.

$$
\begin{aligned}
&\gamma^{\underline{abc}}\gamma_{\underline{defgh}} \\
&= \frac{1}{5!4!2!}\delta_{[\underline{d}}{}^{[\underline{a}}\epsilon_{\underline{efgh}]}{}^{\underline{bc}][5]}\gamma_{[5]} - \frac{1}{3!}\epsilon^{\underline{abc}}{}_{\underline{defgh}}{}^{[3]}\gamma_{[3]} + \frac{1}{12}\delta_{[\underline{d}}{}^{[\underline{a}}\delta_{\underline{e}}{}^{\underline{b}}\gamma_{\underline{fgh}]}{}^{\underline{c}]} - \frac{1}{2}\delta_{[\underline{d}}{}^{\underline{a}}\delta_{\underline{e}}{}^{\underline{b}}\delta_{\underline{f}}{}^{\underline{c}}\gamma_{\underline{gh}]} \\
&= \frac{1}{4!2!}\epsilon^{[4]}{}_{\underline{defgh}}{}^{[\underline{ab}}\gamma^{\underline{c}]}{}_{[4]} - \frac{1}{3!}\epsilon^{\underline{abc}}{}_{\underline{defgh}}{}^{[3]}\gamma_{[3]} + \frac{1}{12}\delta_{[\underline{d}}{}^{[\underline{a}}\delta_{\underline{e}}{}^{\underline{b}}\gamma_{\underline{fgh}]}{}^{\underline{c}]} - \frac{1}{2}\delta_{[\underline{d}}{}^{\underline{a}}\delta_{\underline{e}}{}^{\underline{b}}\delta_{\underline{f}}{}^{\underline{c}}\gamma_{\underline{gh}]} \\
&= \frac{1}{4!4!}\epsilon^{[4]\underline{abc}}{}_{[\underline{defg}}\gamma_{\underline{h}][4]} - \frac{1}{3!}\epsilon^{\underline{abc}}{}_{\underline{defgh}}{}^{[3]}\gamma_{[3]} + \frac{1}{12}\delta_{[\underline{d}}{}^{[\underline{a}}\delta_{\underline{e}}{}^{\underline{b}}\gamma_{\underline{fgh}]}{}^{\underline{c}]} - \frac{1}{2}\delta_{[\underline{d}}{}^{\underline{a}}\delta_{\underline{e}}{}^{\underline{b}}\delta_{\underline{f}}{}^{\underline{c}}\gamma_{\underline{gh}]}
\end{aligned}
$$

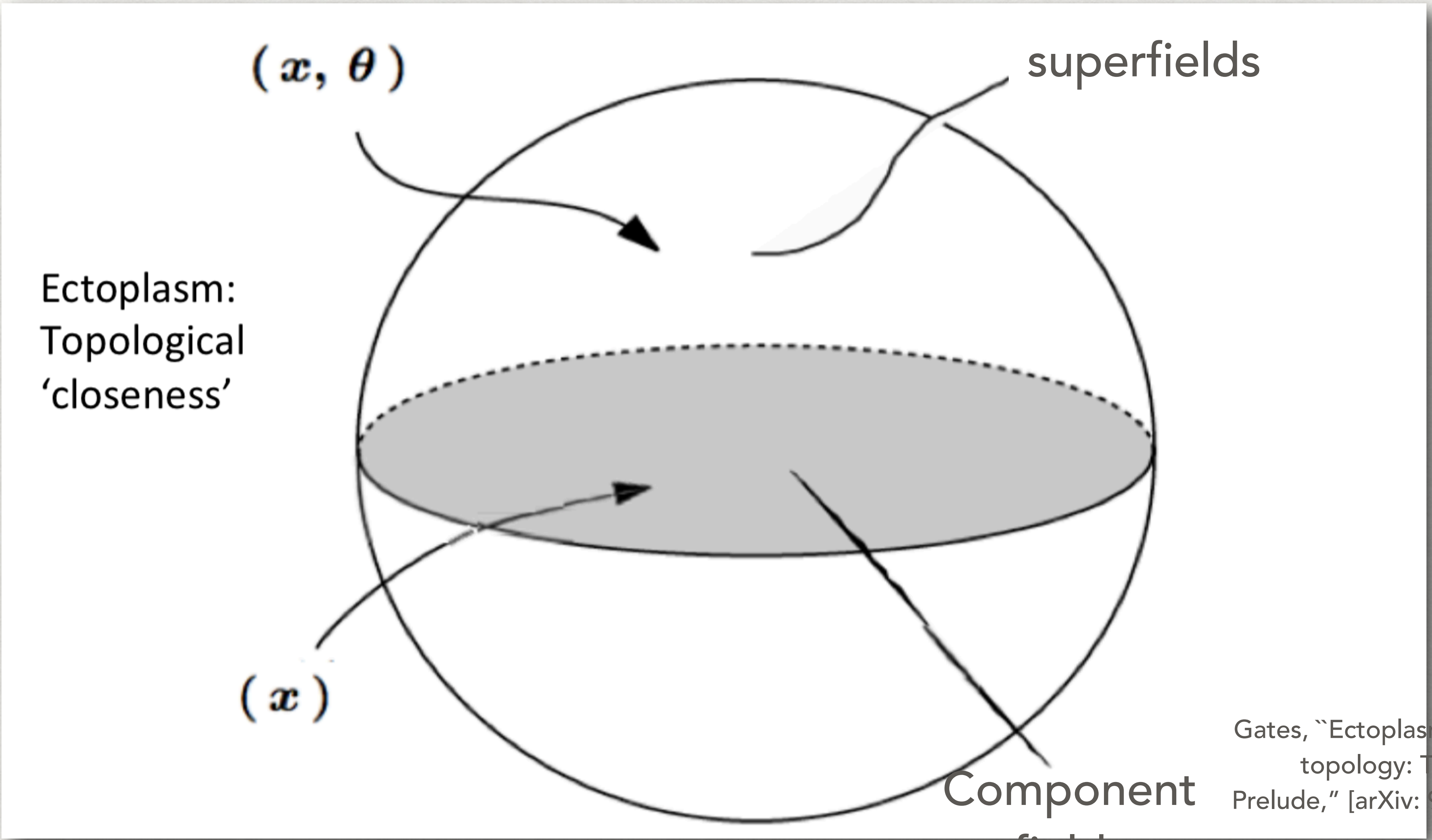
THE ECTOPLASMIC CONJECTURE

"I am thinking about something much more important than bombs. I am thinking about computers"

— John von Neumann

THE ECTOPLASMIC CONJECTURE

A REPRESENTATION OF SUPERSPACE

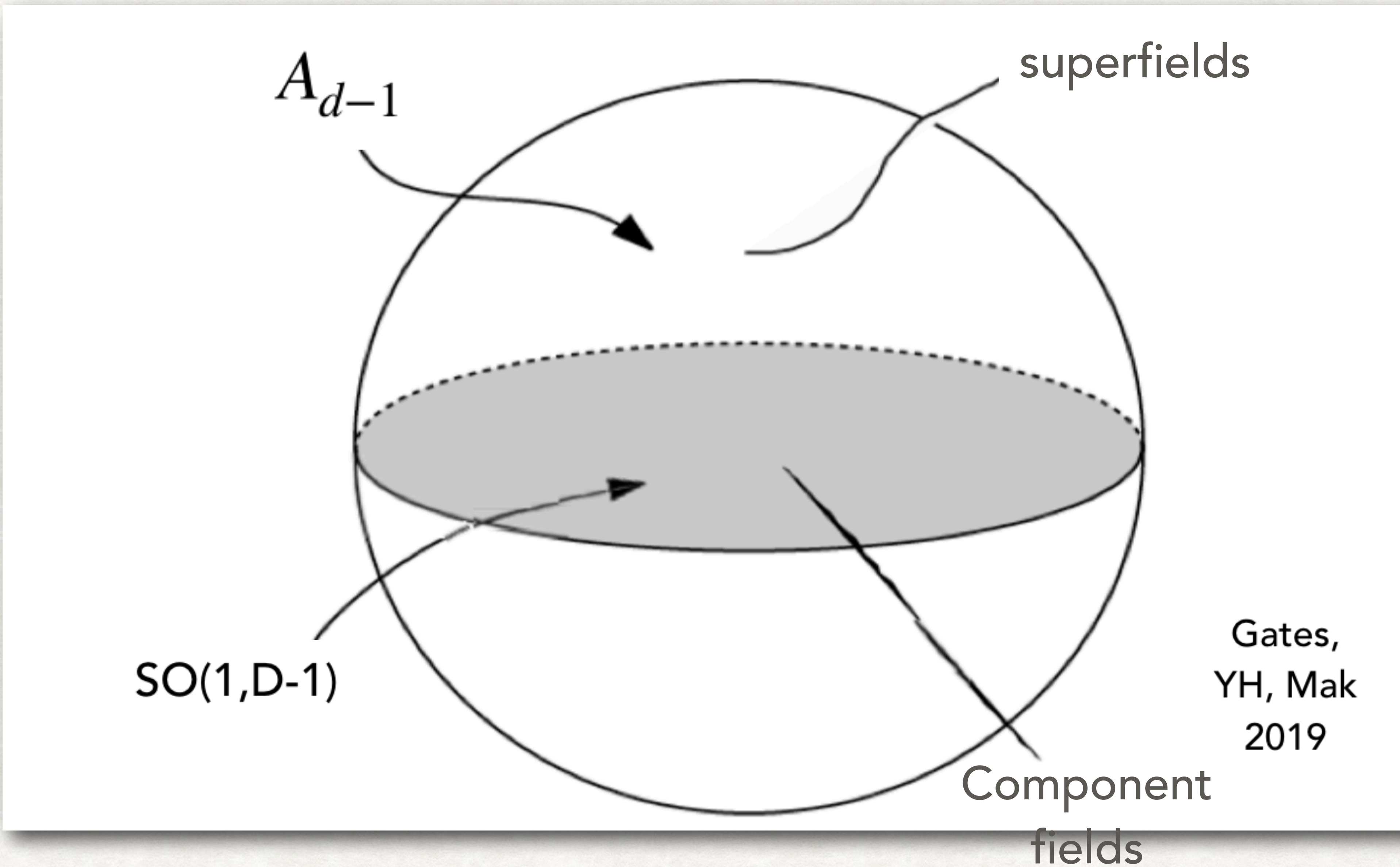


Gates, ``Ectoplasm has no topology: The Prelude," [arXiv: 9709104]

- The volume of the sphere represents the entirety of superspace and the equatorial plane represents the bosonic subspace

THE ECTOPLASMIC CONJECTURE

A REPRESENTATION OF SUPERSPACE



- The volume of the sphere represents the entirety of superspace and the equatorial plane represents the bosonic subspace

BRANCHING RULES

11D SCALAR SUPERFIELD

- Definition: a Branching Rule is a relation between a representation of a Lie algebra $\mathfrak{g}$ and representations of its Lie subalgebra $\mathfrak{h}$

$$\mathfrak{su}(32) \supset \mathfrak{so}(1,10) \quad \Rightarrow \quad \mathcal{R}_{\mathfrak{su}(32)} \xrightarrow{\text{branching rules}} \bigoplus \mathcal{R}_{\mathfrak{so}(1,10)}$$

- Intuition: θ-monomials equivalent to irreps of $\mathfrak{su}(32)$

$$\theta^{\alpha_1} \cdots \theta^{\alpha_n} \quad \Leftrightarrow \quad \underbrace{\{32\} \wedge \ \ldots \ \wedge \{32\}}_{n \text{ times}} \quad \Leftrightarrow \quad \begin{array}{|c|}\hline 32 \\ \hline \vdots \\ \hline 32 \\ -n+1 \\ \hline \end{array}$$

BRANCHING RULES

PROJECTION MATRIX

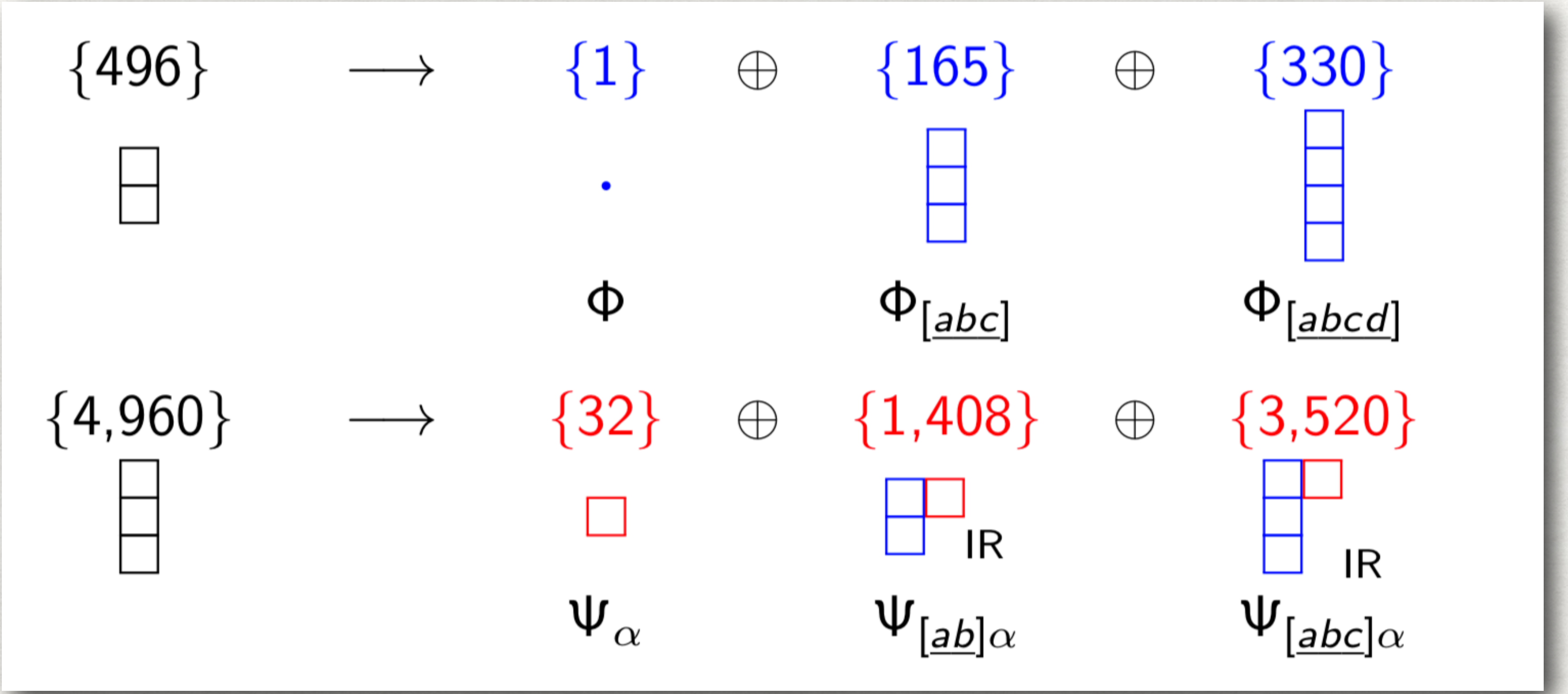
- Branching rules are determined by a single projection matrix
- The projection matrix is fixed by the weight diagrams of a branching rule of them, where weight diagrams can be written down by Cartan matrix of $\mathfrak{g}$ and Dynkin labels
- {32} in $\mathfrak{su}(32)$ = {32} in $\mathfrak{so}(1,10)$ gives

$$P_{su(32)\supset so(1,10)} = \begin{pmatrix} 0&0&0&0&0&0&1&0&0&1&0&0&1&0&1&2&1&0&1&0&0&1&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&2&1&1&2&1&1&2&1&1&0&0&0&1&1&2&1&1&2&1&1&2&1&0&0&0&0 \\ 0&0&1&2&1&0&0&0&0&0&0&0&1&2&3&2&3&2&1&0&0&0&0&0&0&0&1&2&1&0&0 \\ 0&1&0&0&0&1&1&1&0&1&2&2&1&0&0&0&0&0&1&2&2&1&0&1&1&1&0&0&0&1&0 \\ 1&0&1&0&1&0&1&2&3&2&1&0&1&2&1&2&1&2&1&0&1&2&3&2&1&0&1&0&1&0&1 \end{pmatrix}$$

BRANCHING RULES

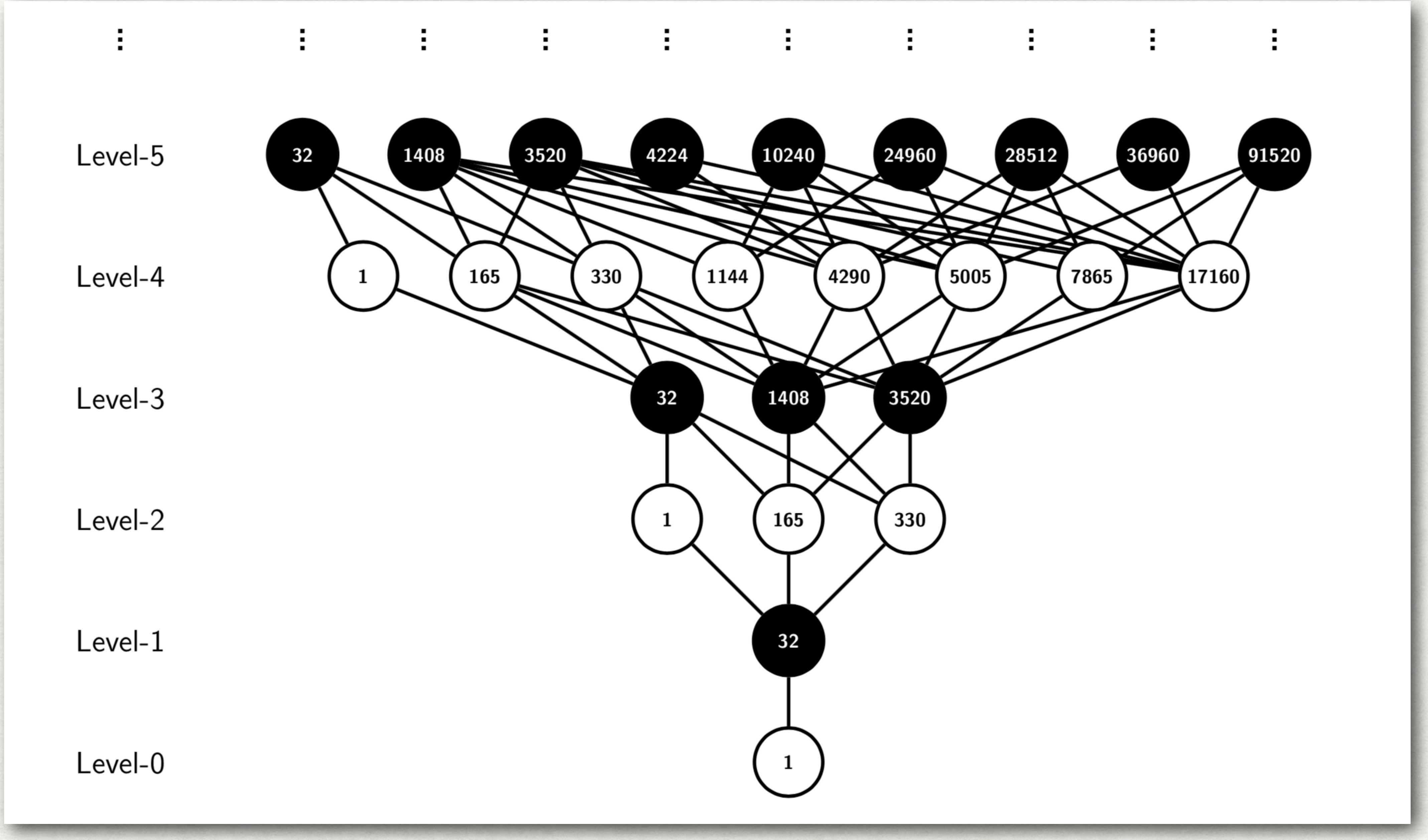
11D SCALAR SUPERFIELD

- Examples



- Dictionary & Graphical Rules in [Gates, YH, Mak, arXiv: 2006.03609] (Bosonic Young Tableaux & Spinorial Young Tableaux introduced)

11D SCALAR SUPERFIELD RESULTS



11D SCALAR SUPERFIELD RESULTS

h

$h_{\mu\nu}$

3-form

- Level-16: $(2)\{1\} \oplus \{11\} \oplus \{65\} \oplus (2)\{165\} \oplus \{275\} \oplus (2)\{330\} \oplus \{462\} \oplus (2)\{935\} \oplus (2)\{1,144\} \oplus \{1,430\} \oplus \{2,717\} \oplus \{3,003\} \oplus (3)\{4,290\} \oplus (2)\{5,005\} \oplus \{7,007\} \oplus (3)\{7,128\} \oplus \{7,150\} \oplus \{7,293\} \oplus (4)\{7,865\} \oplus \{11,583\} \oplus (4)\{15,400\} \oplus \{16,445\} \oplus (5)\{17,160\} \oplus (3)\{22,275\} \oplus (3)\{23,595\} \oplus (2)\{23,595'\} \oplus (2)\{26,520\} \oplus (2)\{28,314\} \oplus (2)\{28,798\} \oplus (3)\{33,033\} \oplus \{35,750\} \oplus (3)\{37,752\} \oplus \{47,190\} \oplus (3)\{57,915\} \oplus (3)\{58,344\} \oplus (3)\{70,070\} \oplus \{72,930\} \oplus (5)\{78,650\} \oplus (2)\{81,510\} \oplus (4)\{85,085\} \oplus \{91,960\} \oplus (2)\{112,200\} \oplus (6)\{117,975\} \oplus (2)\{137,445\} \oplus \{162,162\} \oplus (5)\{175,175\} \oplus (5)\{178,750\} \oplus (2)\{181,545\} \oplus (2)\{182,182\} \oplus (3)\{188,760\} \oplus \{218,295\} \oplus \{235,950\} \oplus \{251,680'\} \oplus (4)\{255,255\} \oplus (2)\{266,266\} \oplus (3)\{268,125\} \oplus (7)\{289,575\} \oplus (4)\{333,234\} \oplus (4)\{382,239\} \oplus (2)\{386,750\} \oplus (2)\{448,305\} \oplus \{490,490\} \oplus (6)\{503,965\} \oplus (3)\{525,525\} \oplus \{526,240\} \oplus \{616,616\} \oplus \{628,320\} \oplus (2)\{650,650\} \oplus \{674,817\} \oplus \{715,715\} \oplus (2)\{722,358\} \oplus (6)\{802,230\} \oplus \{825,825\} \oplus (2)\{862,125\} \oplus (6)\{868,725\} \oplus (4)\{875,160\} \oplus (2)\{948,090\} \oplus (4)\{984,555\} \oplus \{1,002,001\} \oplus (3)\{1,100,385\} \oplus (2)\{1,115,400\} \oplus (2)\{1,123,122\} \oplus \{1,190,112\} \oplus \{1,191,190\} \oplus \{1,245,090\} \oplus (4)\{1,274,130\} \oplus (5)\{1,310,309\} \oplus (2)\{1,412,840\} \oplus (5)\{1,519,375\} \oplus \{1,533,675\} \oplus (4)\{1,673,672\} \oplus (2)\{1,718,496\} \oplus \{1,758,120\} \oplus (3)\{1,786,785\} \oplus \{2,147,145\} \oplus (2)\{2,450,250\} \oplus (2)\{2,571,250\} \oplus \{2,598,960\} \oplus (3)\{2,743,125\} \oplus \{2,858,856\} \oplus \{3,056,625\} \oplus \{3,083,080\} \oplus (4)\{3,128,697\} \oplus \{3,586,440\} \oplus (3)\{3,641,274\} \oplus (2)\{3,792,360\} \oplus \{3,993,990\} \oplus \{4,332,042\} \oplus (4)\{4,506,040\} \oplus (2)\{4,708,704\} \oplus \{4,781,920\} \oplus (6)\{5,214,495\} \oplus (2)\{5,214,495'\} \oplus (2)\{5,651,360\} \oplus \{5,834,400\} \oplus (2)\{6,276,270\} \oplus \{7,468,032\} \oplus (3)\{7,487,480\} \oplus (2)\{7,865,000\} \oplus (3)\{7,900,750\} \oplus \{8,893,500\} \oplus \{9,845,550\} \oplus \{10,696,400'\} \oplus \{10,830,105\} \oplus (2)\{11,981,970\} \oplus \{12,972,960\} \oplus \{14,889,875\} \oplus \{17,606,160\} \oplus \{18,718,700\} \oplus (3)\{20,084,064\} \oplus \{30,604,288\} \oplus \{31,082,480\}$

11D SCALAR SUPERFIELD RESULTS

ψ^{β} $\psi_{\mu}{}^{\alpha}$

- Level-17: $(2)\{32\} \oplus \{320\} \oplus (2)\{1,408\} \oplus \{1,760\} \oplus (3)\{3,520\} \oplus (2)\{4,224\} \oplus \{5,280\} \oplus (3)\{7,040\} \oplus (3)\{10,240\} \oplus (2)\{22,880\} \oplus (3)\{24,960\} \oplus (6)\{28,512\} \oplus (3)\{36,960\} \oplus (4)\{45,056\} \oplus (4)\{45,760\} \oplus \{64,064\} \oplus (6)\{91,520\} \oplus (3)\{128,128\} \oplus (6)\{134,784\} \oplus (3)\{137,280\} \oplus (4)\{147,840\} \oplus (3)\{157,696\} \oplus (5)\{160,160\} \oplus \{160,160'\} \oplus (3)\{183,040\} \oplus (6)\{219,648\} \oplus \{251,680\} \oplus (3)\{264,000\} \oplus (3)\{274,560\} \oplus (3)\{292,864\} \oplus \{302,016\} \oplus \{366,080\} \oplus (2)\{457,600\} \oplus (5)\{480,480\} \oplus (3)\{570,240\} \oplus (7)\{573,440\} \oplus (2)\{672,672\} \oplus (4)\{798,720\} \oplus (5)\{896,896\} \oplus (4)\{901,120\} \oplus (8)\{1,034,880\} \oplus (3)\{1,140,480\} \oplus \{1,171,456\} \oplus \{1,208,064\} \oplus (2)\{1,351,680\} \oplus (3)\{1,425,600\} \oplus (2)\{1,757,184\} \oplus (2)\{1,921,920\} \oplus (3)\{1,936,000\} \oplus (3)\{2,013,440\} \oplus (2)\{2,038,400\} \oplus (5)\{2,114,112\} \oplus (3)\{2,168,320\} \oplus (6)\{2,288,000\} \oplus \{2,342,912\} \oplus (3)\{2,358,720\} \oplus (2)\{2,402,400\} \oplus \{2,446,080\} \oplus (3)\{3,706,560\} \oplus (2)\{3,706,560'\} \oplus (3)\{3,794,560\} \oplus \{4,026,880\} \oplus (6)\{4,212,000\} \oplus (2)\{5,720,000\} \oplus (2)\{5,857,280\} \oplus \{5,930,496\} \oplus (3)\{6,040,320\} \oplus \{6,307,840\} \oplus \{6,864,000\} \oplus (3)\{7,208,960\} \oplus (3)\{8,781,696\} \oplus (3)\{9,123,840\} \oplus \{10,570,560\} \oplus \{10,570,560'\} \oplus (2)\{11,714,560\} \oplus \{11,927,552\} \oplus (2)\{12,390,400\} \oplus (2)\{13,246,464\} \oplus (2)\{13,453,440\} \oplus \{15,375,360\} \oplus \{30,201,600\} \oplus \{33,116,160\} \oplus \{33,554,432\}$

As of now and to the best of our knowledge, no other research group exists that has demonstrated such a capacity to identify the component field spectra in this detail in such systems

11D SCALAR SUPERFIELD RESULTS

Level #	Component Field Count
0	1
1	1
2	3
3	3
4	8
5	9
6	19
7	23
8	49
9	55
10	99
11	106
12	173
13	171
14	247
15	225
16	296

- $N_{Bosonic\ Fields} = 1{,}494$

$$\ldots, h_{\mu\nu}, A_{\mu\nu\rho}, \ldots$$

- $N_{Fermionic\ Fields} = 1{,}186$

$$\ldots, \psi_{\mu}{}^{\alpha}, \ldots$$

BRANCHING RULES

SUMMARY TABLE

D	Branching Rules → component fields	Branching Rules → BYT
11	$A_{31} \supset B_5$	$A_{10} \supset B_5$
10	$A_{15} \supset D_5$	$A_9 \supset D_5$
9	$A_{15} \supset B_4$	$A_8 \supset B_4$
8	$A_{15} \supset D_4$	$A_7 \supset D_4$
7	$A_{15} \supset B_3$	$A_6 \supset B_3$
6	$A_7 \supset D_3 = A_3$	$A_5 \supset D_3$
5	$A_7 \supset B_2 \cong C_2$	$A_4 \supset B_2$
4	$A_3 \supset D_2 \cong A_1 \times A_1$	$A_3 \supset D_2$

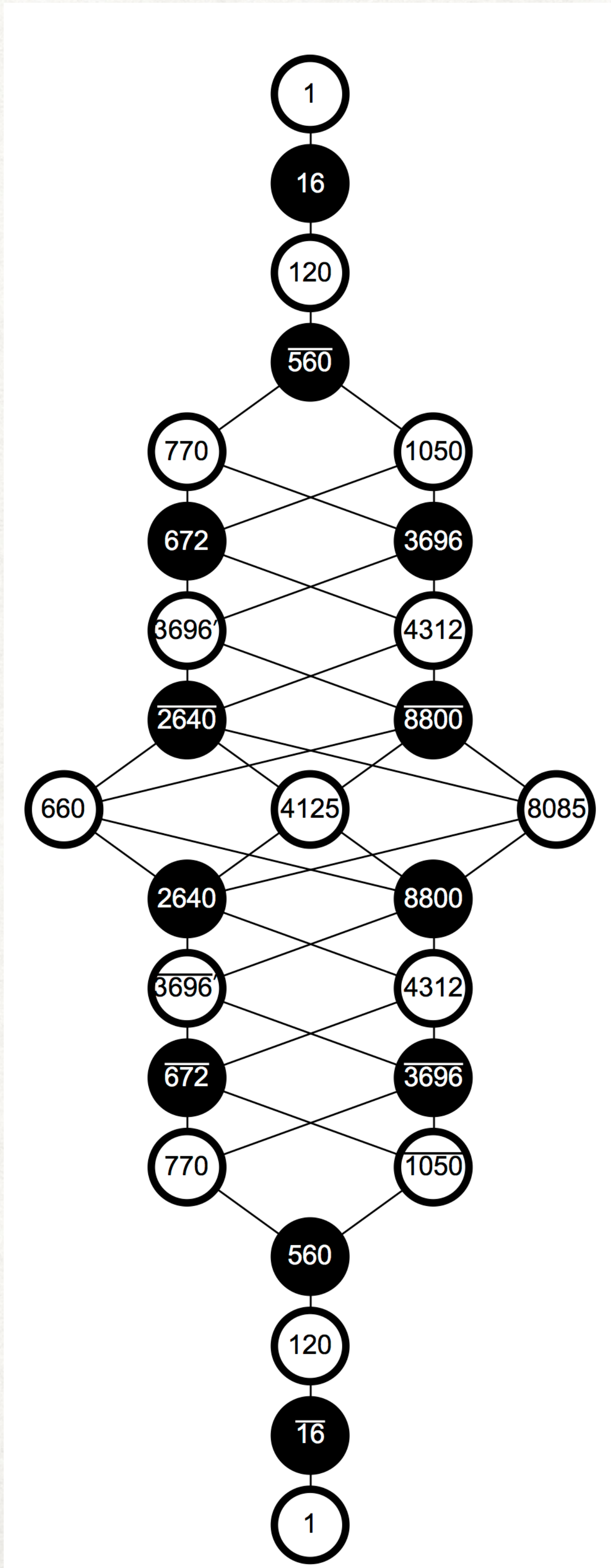
- 10D: [Gates, YH, Mak, JHEP 02 176(2020)]
- 11D: [Gates, YH, Mak, JHEP 09 089(2020)]
- 4D - 9D: [Gates, YH, Mak, JHEP 09 202(2021)]
- Dictionary & Graphical Rules: [Gates, YH, Mak, arXiv: 2006.03609]

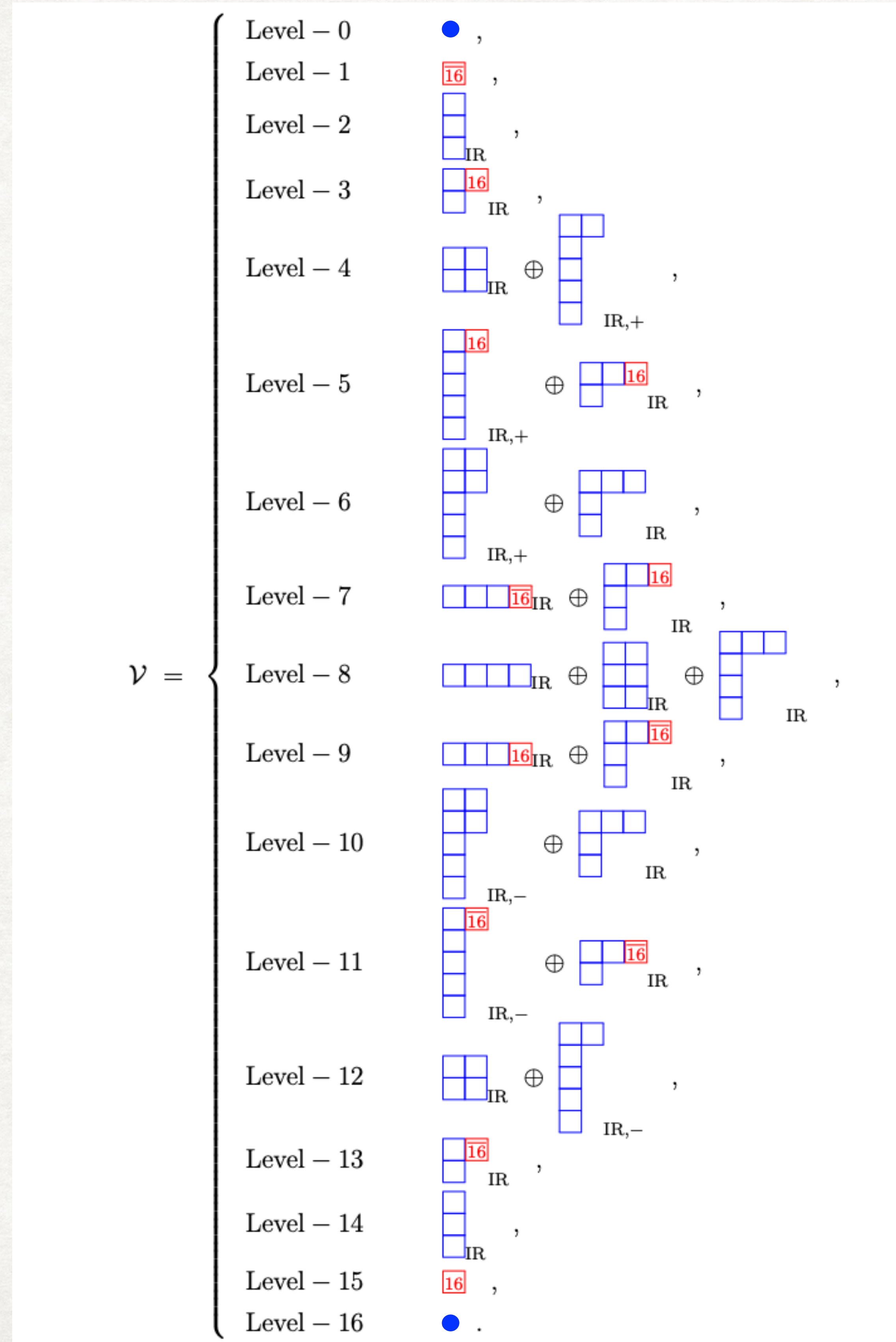
ADYNKRAFIELDS, ADA SCANS, AND 11D SUGRA SURPRISE

"The best way to have a good idea is to have a lot of ideas"

— Linus Pauling

VISIBLE INSIGHTS FROM THE 10D N=1 SCALAR SF





VISIBLE INSIGHTS FROM THE 10D N=1 SCALAR SF

$$\text{Level}-0 \qquad \Phi(x) \quad ,$$

$$\text{Level}-1 \qquad \Psi_{\alpha}(x) \quad ,$$

$$\text{Level}-2 \qquad \Phi_{\{\underline{a}_1\underline{b}_1\underline{c}_1\}}(x) \quad ,$$

$$\text{Level}-3 \qquad \Psi_{\{\underline{a}_1\underline{b}_1\}}{}^{\alpha}(x) \quad ,$$

$$\text{Level}-4 \qquad \Phi_{\{\underline{a}_1\underline{b}_1,\underline{a}_2\underline{b}_2\}}(x) \quad , \qquad \Phi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^+}(x) \quad ,$$

$$\text{Level}-5 \qquad \Psi_{\{\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^+}{}^{\alpha}(x) \quad , \qquad \Psi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\}}{}^{\alpha}(x) \quad ,$$

$$\text{Level}-6 \qquad \Phi_{\{\underline{a}_2\underline{b}_2|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^+}(x) \quad , \qquad \Phi_{\{\underline{a}_2,\underline{a}_3|\underline{a}_1\underline{b}_1\underline{c}_1\}}(x) \quad ,$$

$$\text{Level}-7 \qquad \Psi_{\{\underline{a}_1,\underline{a}_2,\underline{a}_3\}\alpha}(x) \quad , \qquad \Psi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\underline{c}_1\}}{}^{\alpha}(x) \quad ,$$

$$\text{Level}-8 \qquad \Phi_{\{\underline{a}_1,\underline{a}_2,\underline{a}_3,\underline{a}_4\}}(x) \quad , \qquad \Phi_{\{\underline{a}_1\underline{b}_1\underline{c}_1,\underline{a}_2\underline{b}_2\underline{c}_2\}}(x) \quad , \qquad \Phi_{\{\underline{a}_2,\underline{a}_3|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\}}(x) \quad ,$$

$$\text{Level}-9 \qquad \Psi_{\{\underline{a}_1,\underline{a}_2,\underline{a}_3\}}{}^{\alpha}(x) \quad , \qquad \Psi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\underline{c}_1\}\alpha}(x) \quad ,$$

$$\text{Level}-10 \qquad \Phi_{\{\underline{a}_2\underline{b}_2|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^-}(x) \quad , \qquad \widehat{\Phi}_{\{\underline{a}_2,\underline{a}_3|\underline{a}_1\underline{b}_1\underline{c}_1\}}(x) \quad ,$$

$$\text{Level}-11 \qquad \Psi_{\{\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^-\alpha}(x) \quad , \qquad \Psi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\}\alpha}(x) \quad ,$$

$$\text{Level}-12 \qquad \widehat{\Phi}_{\{\underline{a}_1\underline{b}_1,\underline{a}_2\underline{b}_2\}}(x) \quad , \qquad \Phi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^-}(x) \quad ,$$

$$\text{Level}-13 \qquad \Psi_{\{\underline{a}_1\underline{b}_1\}\alpha}(x) \quad ,$$

$$\text{Level}-14 \qquad \widehat{\Phi}_{\{\underline{a}_1\underline{b}_1\underline{c}_1\}}(x) \quad ,$$

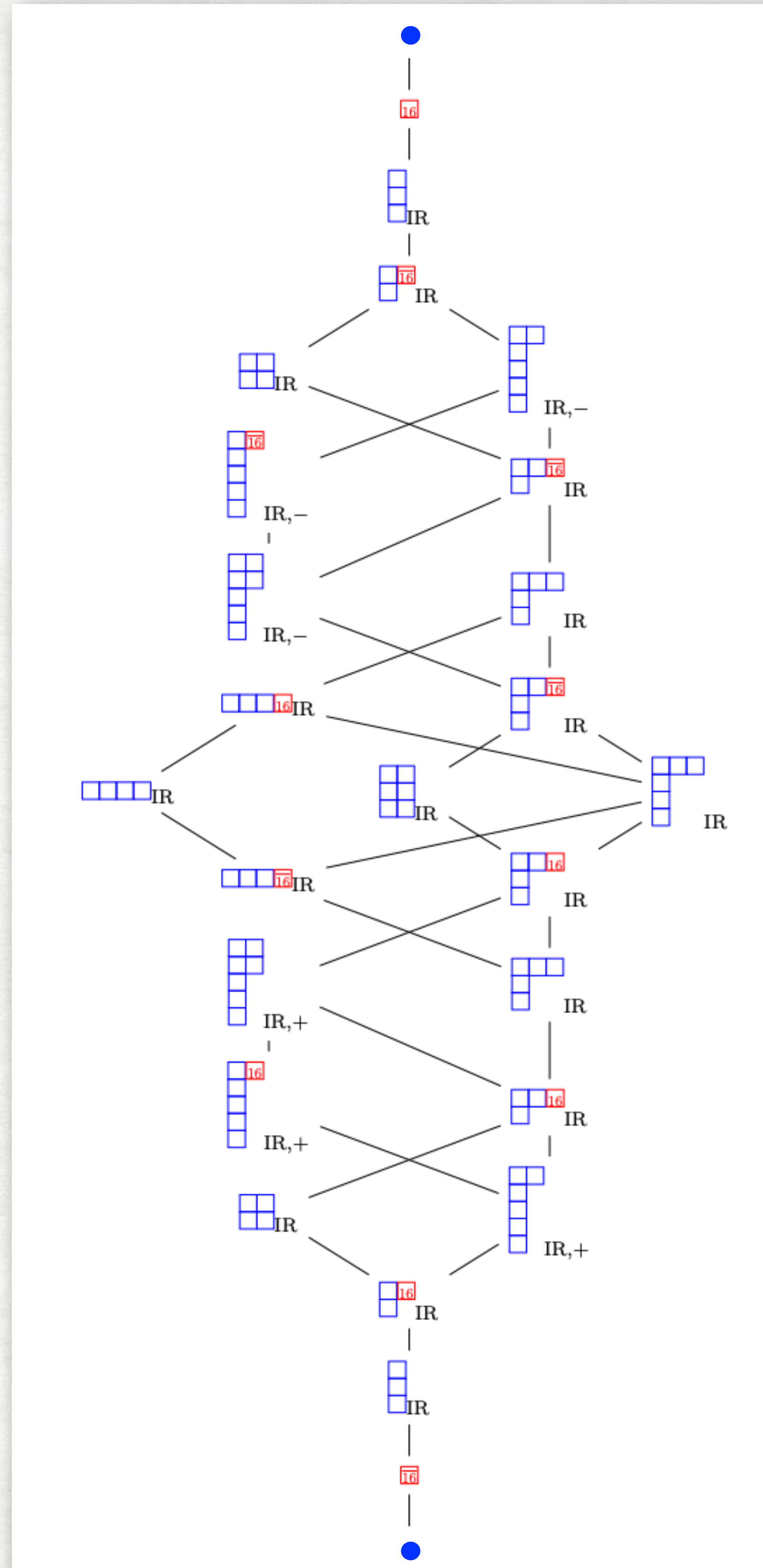
$$\text{Level}-15 \qquad \Psi^{\alpha}(x) \quad ,$$

$$\text{Level}-16 \qquad \widehat{\Phi}(x) \quad .$$

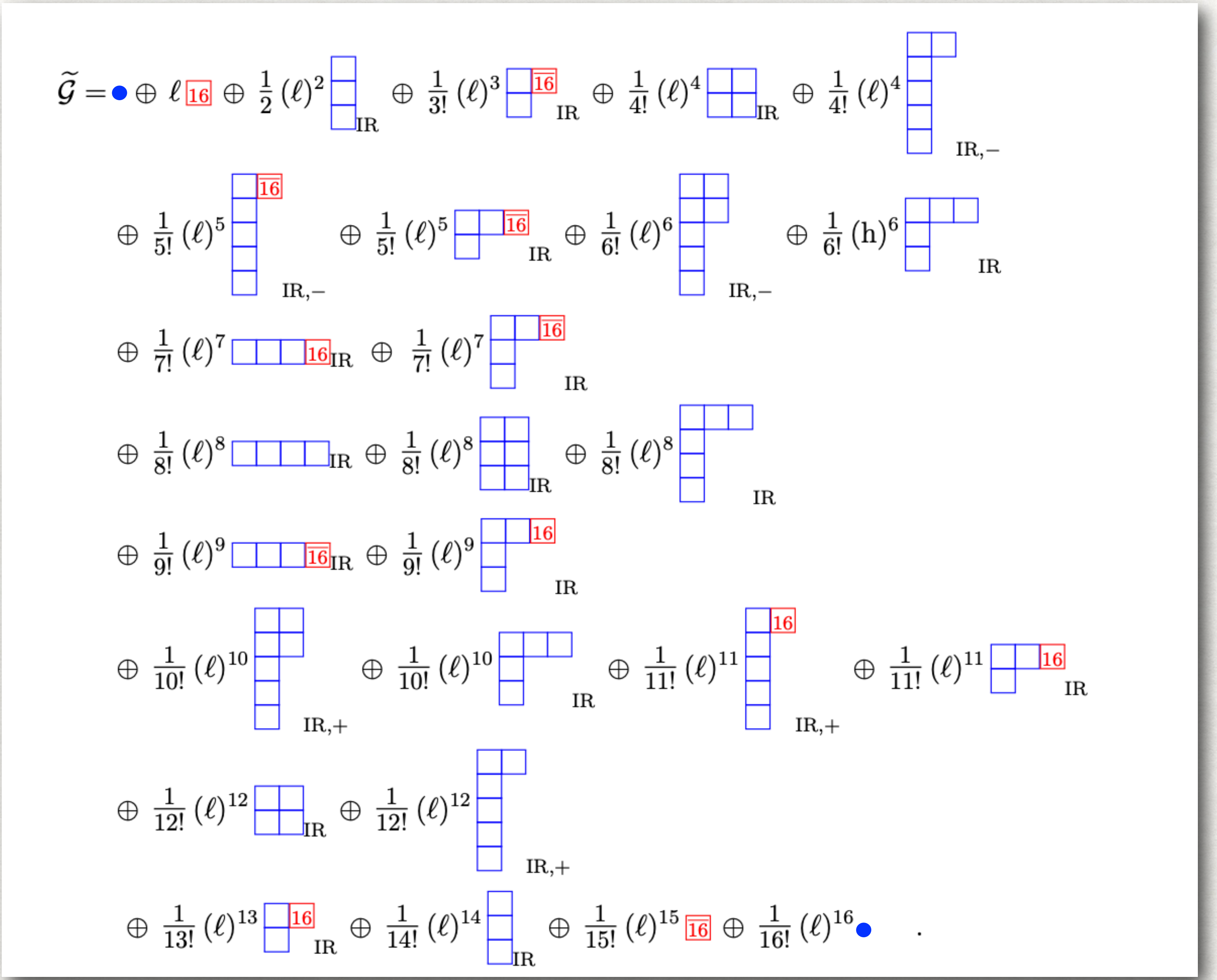
ADYNKRAS

- 10D, N=1 Adynkra graph in Dynkin labels / Young Tableaux forms

- Can we define a new formalism in which θ-monomials are replaced by Young Tableau?



THE 10D, N=1 SUPERFIELD GENOME



$[\ell] = [\theta]$

10D, N=1 ADYNKRAFIELDS

$$
\begin{aligned}
\widehat{\mathcal{G}}(x) \;=\; & \Phi(x) + \ell\, \begin{ytableau} 16 \end{ytableau}\, \Psi_{\alpha}(x) + \frac{1}{2}(\ell)^2 \ydiagram{1,1,1}_{\mathrm{IR}}\, \Phi_{\{\underline{a}_1\underline{b}_1\underline{c}_1\}}(x) + \frac{1}{3!}(\ell)^3 \begin{ytableau} \; & \overline{16} \\ \; \end{ytableau}_{\mathrm{IR}}\, \Psi_{\{\underline{a}_1\underline{b}_1\}}{}^{\alpha}(x) \\
& + \frac{1}{4!}(\ell)^4 \ydiagram{2,2}_{\mathrm{IR}}\, \Phi_{\{\underline{a}_1\underline{b}_1,\underline{a}_2\underline{b}_2\}}(x) + \frac{1}{4!}(\ell)^4 \ydiagram{2,1,1,1,1}_{\mathrm{IR},-}\, \Phi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^+}(x) \\
& + \frac{1}{5!}(\ell)^5 \begin{ytableau} \; & \overline{16} \\ \; \\ \; \\ \; \\ \; \end{ytableau}_{\mathrm{IR},-}\, \Psi_{\{\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^+}{}^{\alpha}(x) + \frac{1}{5!}(\ell)^5 \begin{ytableau} \; & \; & \overline{16} \\ \; \end{ytableau}_{\mathrm{IR}}\, \Psi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\}}{}^{\alpha}(x) \\
& + \frac{1}{6!}(\ell)^6 \ydiagram{2,2,1,1,1}_{\mathrm{IR},-}\, \Phi_{\{\underline{a}_2\underline{b}_2|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^+}(x) + \frac{1}{6!}(\ell)^6 \ydiagram{3,1,1}_{\mathrm{IR}}\, \Phi_{\{\underline{a}_2,\underline{a}_3|\underline{a}_1\underline{b}_1\underline{c}_1\}}(x) \\
& + \frac{1}{7!}(\ell)^7 \begin{ytableau} \; & \; & \; & 16 \end{ytableau}_{\mathrm{IR}}\, \Psi_{\{\underline{a}_1,\underline{a}_2,\underline{a}_3\}\alpha}(x) + \frac{1}{7!}(\ell)^7 \begin{ytableau} \; & \; & \overline{16} \\ \; \\ \; \end{ytableau}_{\mathrm{IR}}\, \Psi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\underline{c}_1\}}{}^{\alpha}(x) \\
& + \frac{1}{8!}(\ell)^8 \ydiagram{4}_{\mathrm{IR}}\, \Phi_{\{\underline{a}_1,\underline{a}_2,\underline{a}_3,\underline{a}_4\}}(x) + \frac{1}{8!}(\ell)^8 \ydiagram{2,2,2}_{\mathrm{IR}}\, \Phi_{\{\underline{a}_1\underline{b}_1\underline{c}_1,\underline{a}_2\underline{b}_2\underline{c}_2\}}(x) \\
& + \frac{1}{8!}(\ell)^8 \ydiagram{3,1,1,1}_{\mathrm{IR}}\, \Phi_{\{\underline{a}_2,\underline{a}_3|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\}}(x) + \mathcal{O}((l)^9)
\end{aligned}
$$

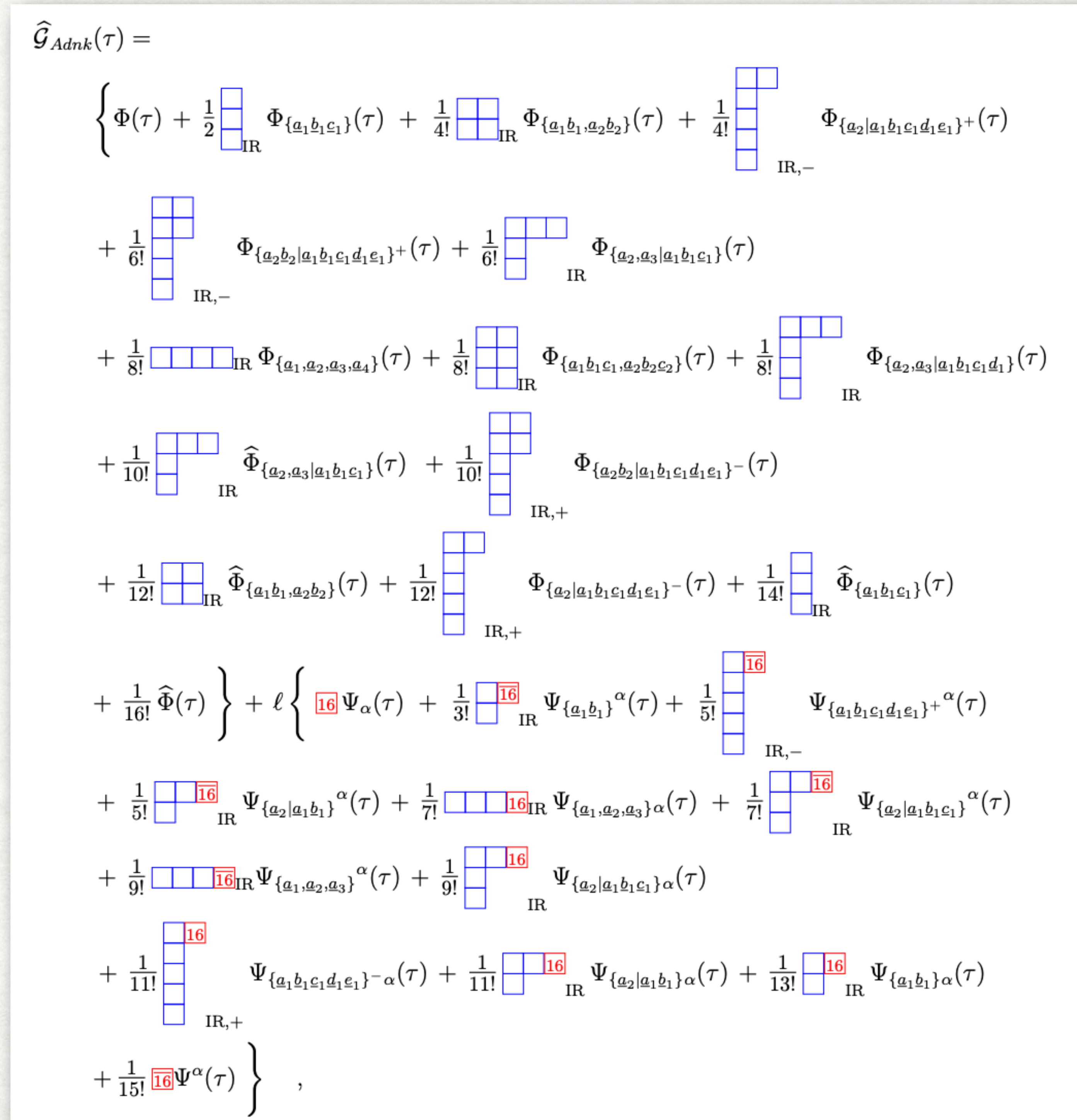
FROM 10D, N=1 BACK TO 1D, N=16

- 10D, N=1 → 1D, N=16
- We can take a limit:
  - all of the field variables depend solely on a time-like coordinate **τ**
  - impose the condition that $(\ell)^2 = 1$

FROM 10D, N=1 BACK TO 1D,N=16

- It contains 32,768 bosons and 32,768 fermions.

- It also contains the information associated with the Lorentz representations (via the YT's) of the original 10D, N = 1 scalar supermultiplet for which it is the hologram



ADYNKRA DIGITAL ANALYSIS SCANS

BREITENLOHNER APPROACH

- Idea: attach bosonic and spinor indices on the scalar superfield and look for components that occur onshell [Gates, YH, Mak, JHEP 03 (2021) 074]

- The first off-shell description of $4\mathrm{D}, \mathcal{N}=1$ supergravity was carried out by Breitenlohner in 1977: start with the component fields of the WZ gauge $4\mathrm{D}, \mathcal{N}=1$ vector supermultiplet

$$
\begin{aligned}
\mathrm{D}_\alpha v_{\underline{a}} &= (\gamma_{\underline{a}})_\alpha{}^\beta \lambda_\beta \quad , \\
\mathrm{D}_\alpha \lambda_\beta &= -\, i\, \tfrac{1}{4}([\gamma^{\underline{a}}, \gamma^{\underline{b}}])_{\alpha\beta} (\partial_{\underline{a}} v_{\underline{b}} - \partial_{\underline{b}} v_{\underline{a}}) + (\gamma^5)_{\alpha\beta}\, \mathrm{d} \quad , \\
\mathrm{D}_\alpha\, \mathrm{d} &= i\, (\gamma^5 \gamma^{\underline{a}})_\alpha{}^\beta\, \partial_{\underline{a}} \lambda_\beta \quad ,
\end{aligned}
$$

- Do a series of replacements of the fields

$$
v_{\underline{a}} \to h_{\underline{a}\,\underline{b}} \quad , \quad \lambda_\beta \to \psi_{\underline{b}\,\beta} \quad , \quad \mathrm{d} \to A_{\underline{b}}
$$

11D SUPERGRAVITY SURPRISE

PPOINCARE VIELBEIN & GRAVITINO

- Decompositions of the inverse frame and gravitino fields in 11D yield

$$
\begin{array}{llll}
e_{\underline{a}}{}^{m} = & \{h_{(\underline{ab})} + & \eta_{\underline{ab}} h + & h_{[\underline{ab}]}\}\,\eta^{\underline{bm}} \\
\{121\} & \{65\} & \{1\} & \{55\}
\end{array}
$$

- $h_{(ab)}$ is the conformal graviton, h is the trace, and $h_{[ab]}$ is the two form

$$
\begin{array}{lll}
\tilde{\psi}_{\underline{a}}{}^{\alpha} & = \psi_{\underline{a}}{}^{\alpha} & - \dfrac{1}{11}(\gamma_{\underline{a}})^{\alpha\beta}\psi_{\beta} \\
\{352\} & \{320\} & \{32\}
\end{array}
$$

- $\psi_a{}^\alpha$ is the conformal gravitino and ψ_β is the γ-trace

11D SUPERGRAVITY SURPRISE

PREPOTENTIAL CANDIDATES

- Semi-prepotential candidate: $\mathcal{V} = \mathrm{D}^{\alpha}\,\mathcal{V}_{\alpha}$

Physical Component	Irrep	Level
graviton $h_{\underline{ab}}$	$\{65\}$, $\{1\}$	16
gravitino $\psi_{\underline{a}}{}^{\beta}$	$\{320\}$, $\{32\}$	17
3-form $B_{[3]}$	$\{165\}$	16

- Prepotential candidate: $\mathcal{V}_{\alpha}$
  - Contains 2-form at level-17 $\Rightarrow$ Poincare vielbein
-

SUSY HOLOGRAPHY CONJECTURE

"Living is worthwhile if one can contribute in some small way to this endless chain of progress."

— Paul A.M. Dirac

SUSY HOLOGRAPHY CONJECTURE

IDEA

- SUSY Holography Conjecture: reduce higher dimensional supersymmetric models to 1D, 1D models encode the structure of higher dimensional models.

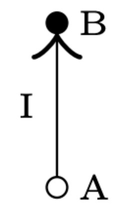
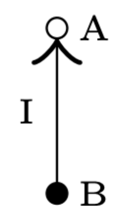
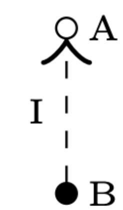
- Key object: adinkra — a graphical representation of 1D, N-extended SUSY algebra [Faux, Gates, 2005]

- 1D N-extended Super-Poincaré (1|N) generated by $\{Q_I, Q_J\} = 2i\delta_{IJ}\partial_\tau, \ [Q_I, \partial_\tau] = [\partial_\tau, \partial_\tau] = 0$

- Off-shell supermultiplet: $A, B = 1, \ldots, d; \ I = 1, \ldots, N; \ c = \pm 1;$ and $\lambda = 0,1$

$$Q_I\,\phi_A(\tau) = c\partial_\tau^\lambda \psi_B(\tau),$$
$$Q_I\,\psi_B(\tau) = \frac{i}{c}\partial_\tau^{1-\lambda}\phi_A(\tau),$$

DEFINITION OF THE ADINKRA

Action of Q_I	Adinkra	Action of Q_I	Adinkra
$Q_I \begin{bmatrix} \psi_B \\ \phi_A \end{bmatrix} = \begin{bmatrix} i\dot{\phi}_A \\ \psi_B \end{bmatrix}$		$Q_I \begin{bmatrix} \psi_B \\ \phi_A \end{bmatrix} = \begin{bmatrix} -i\dot{\phi}_A \\ -\psi_B \end{bmatrix}$	
$Q_I \begin{bmatrix} \phi_A \\ \psi_B \end{bmatrix} = \begin{bmatrix} i\dot{\psi}_B \\ \phi_A \end{bmatrix}$		$Q_I \begin{bmatrix} \phi_A \\ \psi_B \end{bmatrix} = \begin{bmatrix} -i\dot{\psi}_B \\ -\phi_A \end{bmatrix}$	

[Doran, Iga, Kostiuk, Landweber, Mendez-Diez, 2013]

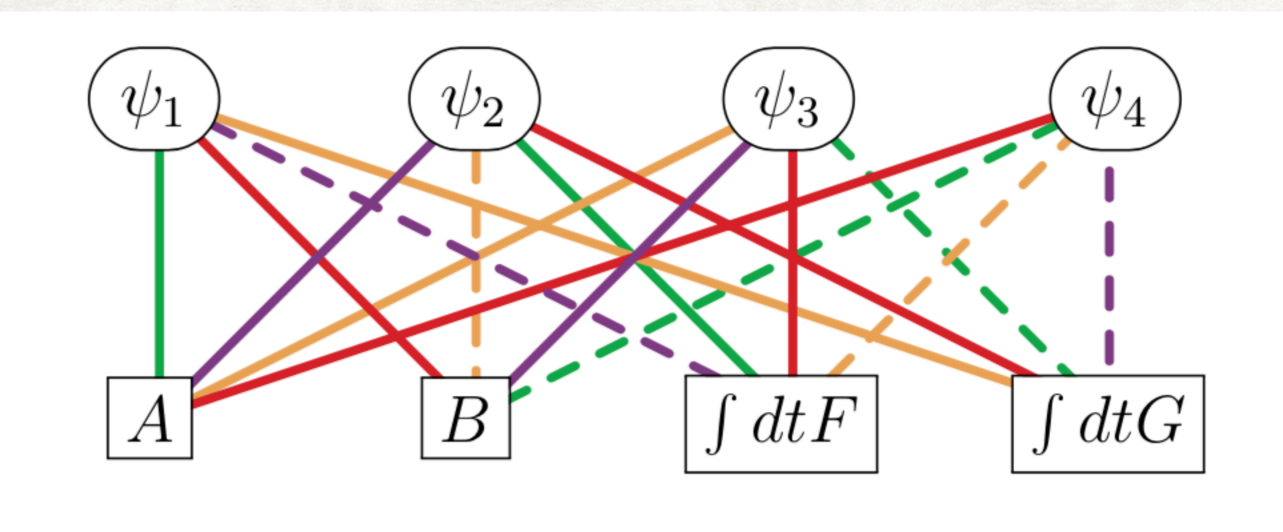
- Each white vertex = bosonic component field/its time derivative
- Each black vertex = fermionic component field/its time derivative
- Edges colored by color I (Q_I)
- Edge is <u>oriented</u> : white -> black if $\lambda = 0$; black -> white if $\lambda = 1$
- Edge is dashed if $c = -1$; solid if $c = 1$

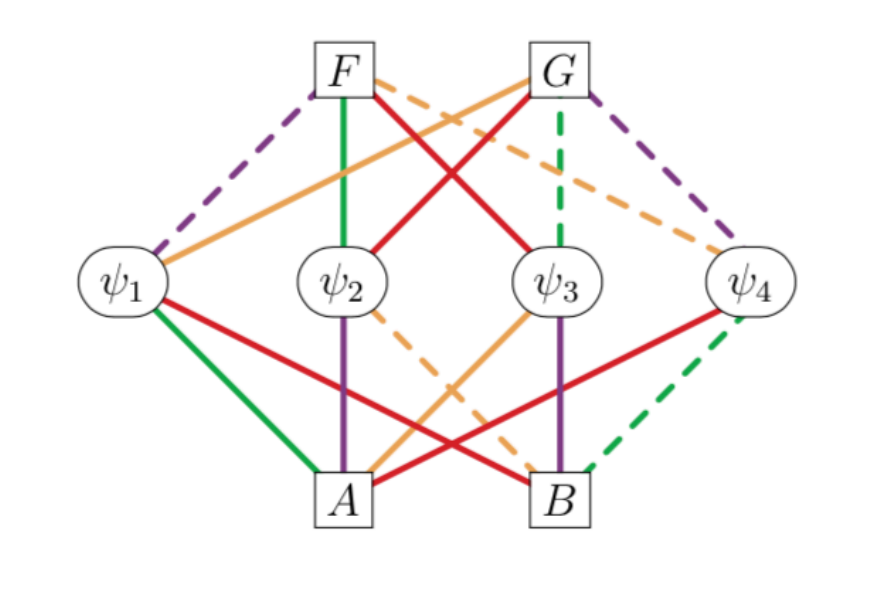
1D, N=4 EXAMPLE: 4D, N=1 CHIRAL

- SUSY transformation laws for 4D, N=1 Chiral supermultiplet:

$$\mathrm{D}_a A = \psi_a \quad , \quad \mathrm{D}_a B = i(\gamma^5)_a{}^b \psi_b \quad , \quad \mathrm{D}_a F = (\gamma^\mu)_a{}^b \partial_\mu \psi_b \quad , \quad \mathrm{D}_a G = i(\gamma^5 \gamma^\mu)_a{}^b \partial_\mu \psi_b \quad ,$$
$$\mathrm{D}_a \psi_b = i(\gamma^\mu)_{ab} \partial_\mu A - (\gamma^5 \gamma^\mu)_{ab} \partial_\mu B - i C_{ab} F + (\gamma^5)_{ab} G \quad .$$

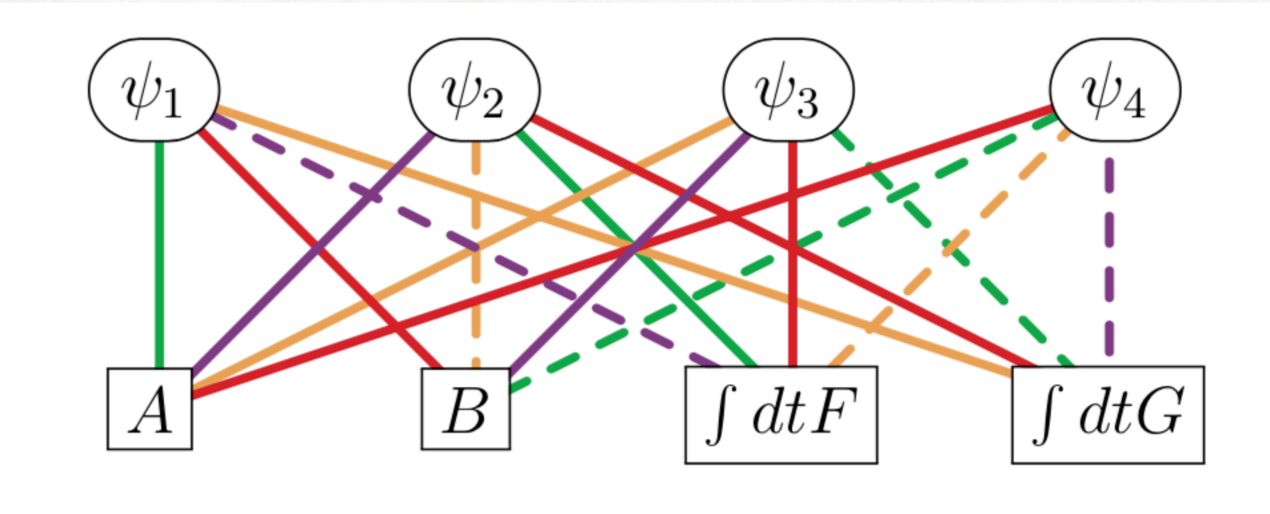
- Restrict the functions only to be dependent on the t-coordinate $\Rightarrow$ 4D, N=1 Chiral multiplet on the 0-Brane.





[Gates, YH, Stiffler, 2019, arXiv: 1904.01738]

GRAPHS AS NETWORKS



$$(L_1)_{i\hat{k}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix} , \quad (L_2)_{i\hat{k}} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(L_3)_{i\hat{k}} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{pmatrix} , \quad (L_4)_{i\hat{k}} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} .$$

- Adinkra (network) ⇔ L/R adjacent matrices
- SUSY transformation laws encoded by valise adinkras can be described by $\quad \mathrm{D_I}\Phi_i = i(\mathrm{L_I})_{i\hat{k}}\Psi_{\hat{k}} \quad , \quad \mathrm{D_I}\Psi_{\hat{k}} = i(\mathrm{R_I})_{\hat{k}i}\Phi_i \quad , \quad \mathrm{R_I} = (\mathrm{L_I})^{\mathrm{T}}$
- N $\mathrm{L_I}$ and N $\mathrm{R_I}$ matrices satsify the so-called Garden Algebra *GR*(d,N): $\quad \mathrm{L_I R_J} + \mathrm{L_J R_I} = 2\delta_{\mathrm{IJ}}\mathrm{I_d} \quad , \quad \mathrm{R_I L_J} + \mathrm{R_J L_I} = 2\delta_{\mathrm{IJ}}\mathrm{I_d}$

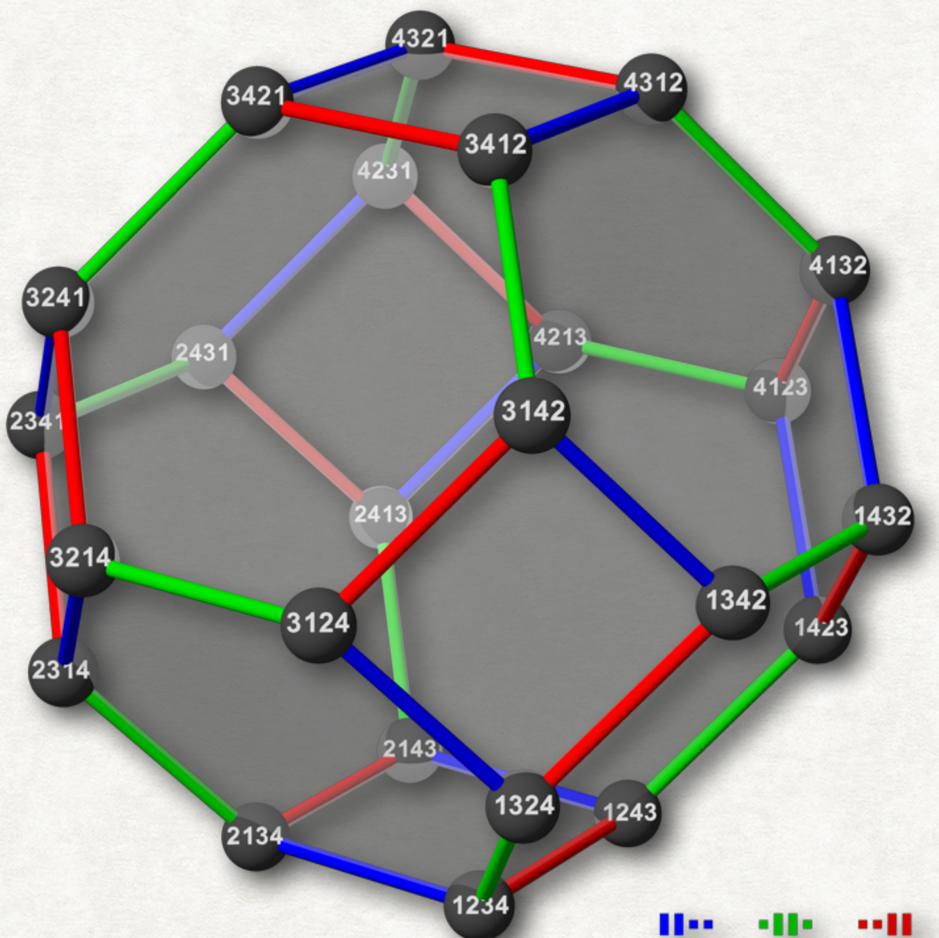
N=4: TOTAL # & CLASSIFICATIONS

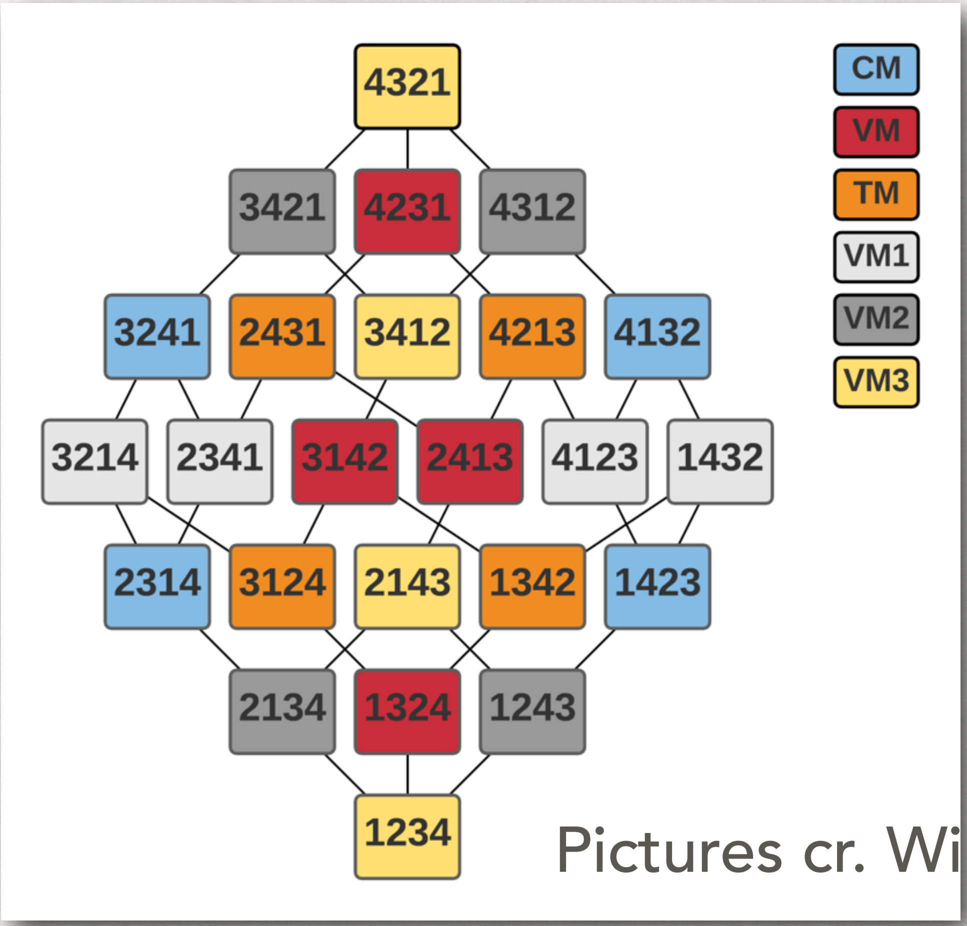
- What's the total number of all possible N=4 valise adinkras?
  - signed permutations of colors and bosons from two quaternion seed adinkras $(\mathrm{L_I})_{\mathrm{i}\hat{\mathrm{j}}} = (\mathrm{BC_4})_{\mathrm{ik}}(\mathrm{BC_3})_{\mathrm{IJ}}(\mathrm{L_J^{seed}})_{\mathrm{k}\hat{\mathrm{j}}}$
  - counting = $2 \times BC_4(\text{boson}) \times BC_4(\text{color})/\text{Isometries} = 36{,}864$ [Gates, Iga, Kang, Korotkikh, Stiffler, 2019]
  - Isometries: sign double counting [e.g. $(1\bar{3}) = -(\bar{2}4)$] $\times$ Klein-four subgroup $\leftrightarrow \{2 \times 4 = 8\}$
  - Classifications? Isomorphism-equivalence classes: [Gates, YH, Stiffler, 2019], [Gates, YH, Stiffler, 2020]

N=4: PERMUTOHEDRON

- What mathematical structure secretly contains information from higher dimensions?

- Toy models: visualizing S_4 (permutohedron)





Pictures cr. Wikipedia

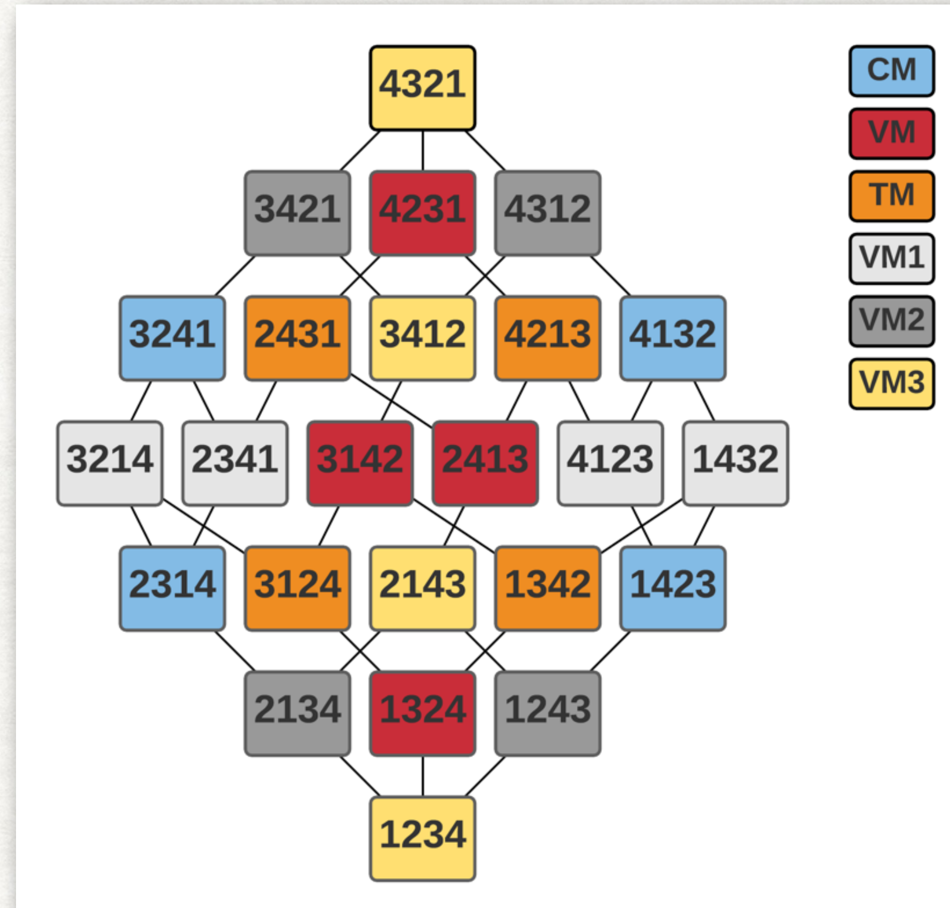
- Consider 4D, N=1 to 1D, N=4, a dissection of S_4 is required

N=4: HOPPING OPERATORS

- Q: what operators connect all the states within the specified SUSY quartets? — “Hopping” Operators

- A: Klein-four subgroup (“Klein’s Vierergruppe)

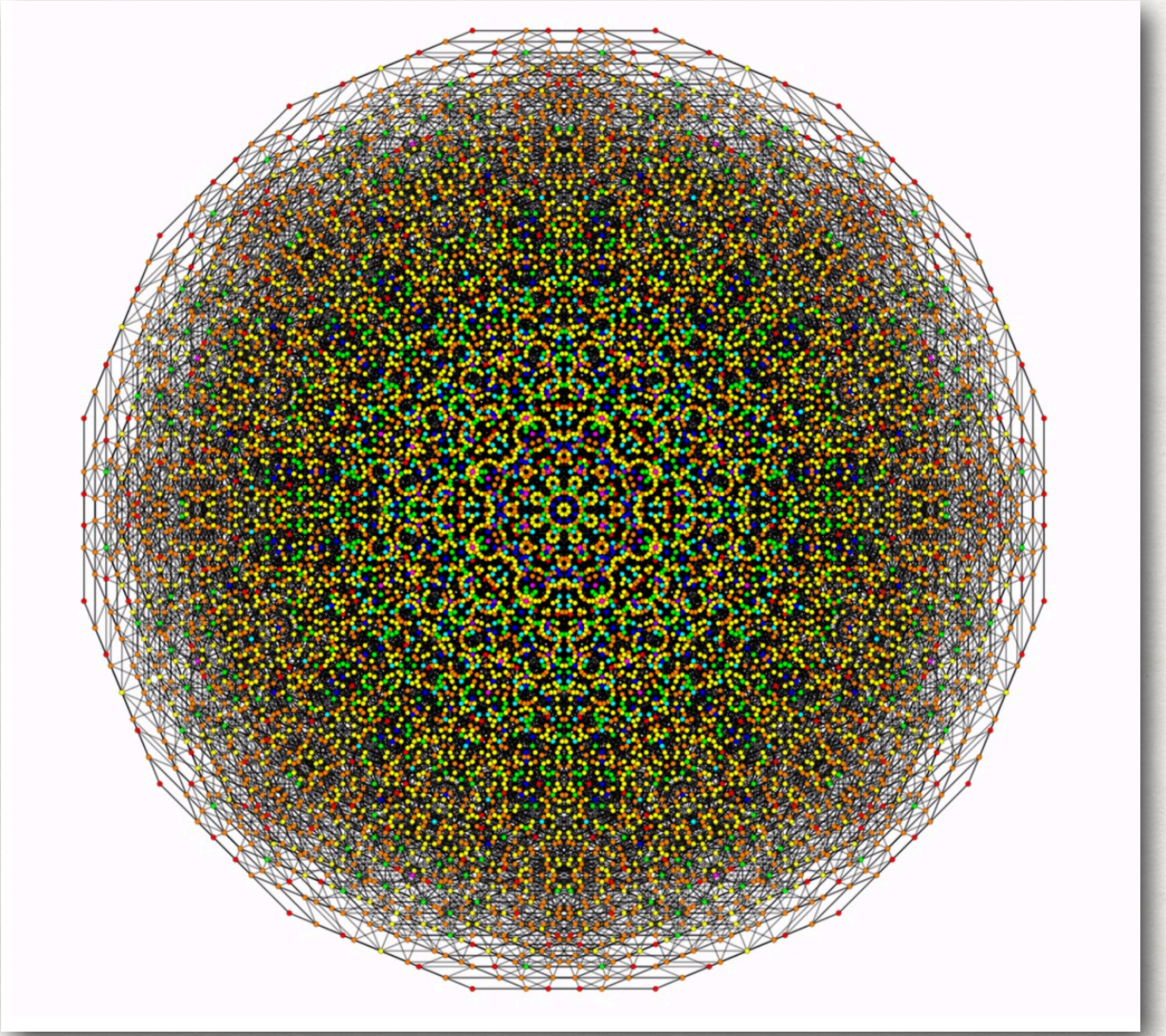
$$
\begin{aligned}
\mathscr{H}_1 &= () \\
\mathscr{H}_2 &= (12)(34) \\
\mathscr{H}_3 &= (23)(12)(34)(23) \\
\mathscr{H}_4 &= (23)(12)(34)(23)(12)(34)
\end{aligned}
$$



[Cianciara, Gates, YH, Kirk, JHEP 05, 077(2021)]

NEXT STOP: N=8?

- N=8: 4D, N = 2 SUSY & the 40,320 Nodes & 141,120 Edges Of the “Omnitruncated 7-simplex”



Also called Hexipentisteriruncicantitruncated 7-simplex. Picture is obtained from Wikipedia

CONCLUSIONS & OUTLOOK

"The most effective way to do it, is to do it."

— Amelia Earhart

CONCLUSIONS

- Our work substantially lowers the computational costs of determining how to embed a set of component fields within a Salam-Strathdee superfield with no additional constraints.
- These embeddings are constructed without information from an off-shell component formulation for the first time
- Our work leads to a formalism demonstrating a manefest linear realization of the Lorentz group
- A proposal to identify possible supergravity prepotential candidates was presented
- These newly developed techniques can also be applied to create unprecedented understanding of M-Theory and F-Theory as relates to their SG limits

OPEN QUESTIONS

- How to determine the complete sets of SUSY transformations for these fields?
  - Part of the information is encoded in the adynkra graphs as discussed in [Gates, YH, Mak, arXiv: 2006.03609]
- Explicit SUSY covariant derivative operation to adynkrafields
- The Salam-Strathdee superfield superconformal gauge group of supergravity
  - Starting point: a re-imaging of adynkrafield formulation of 4D, N = 1 supergravity

THANK YOU!!

"The object of pure Physics is the unfolding of the laws of the intelligible world; the object of pure Mathematics that of unfolding the laws of human intelligence."

— J. J. Sylvester

10D IRREDUCIBLE BOSONIC YOUNG TABLEAUX

$\boxed{a_1}\boxed{a_2} = \{55\}$

$\boxed{a_1}\boxed{a_2}_{\text{IR}} = \{54\}$

$[2, 0, 0, 0, 0]$

$$
\begin{aligned}
\tilde{h}_{\underline{ab}} &= h_{\underline{ab}} + \eta_{\underline{ab}}\, h \\
\{55\} &= \{54\} \oplus \{1\}
\end{aligned}
$$

- Consider the Projection Matrix for $\mathfrak{su}(10) \supset \mathfrak{so}(10)$:

$$
P_{\mathfrak{su}(10)\supset\mathfrak{so}(10)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \end{pmatrix}
$$

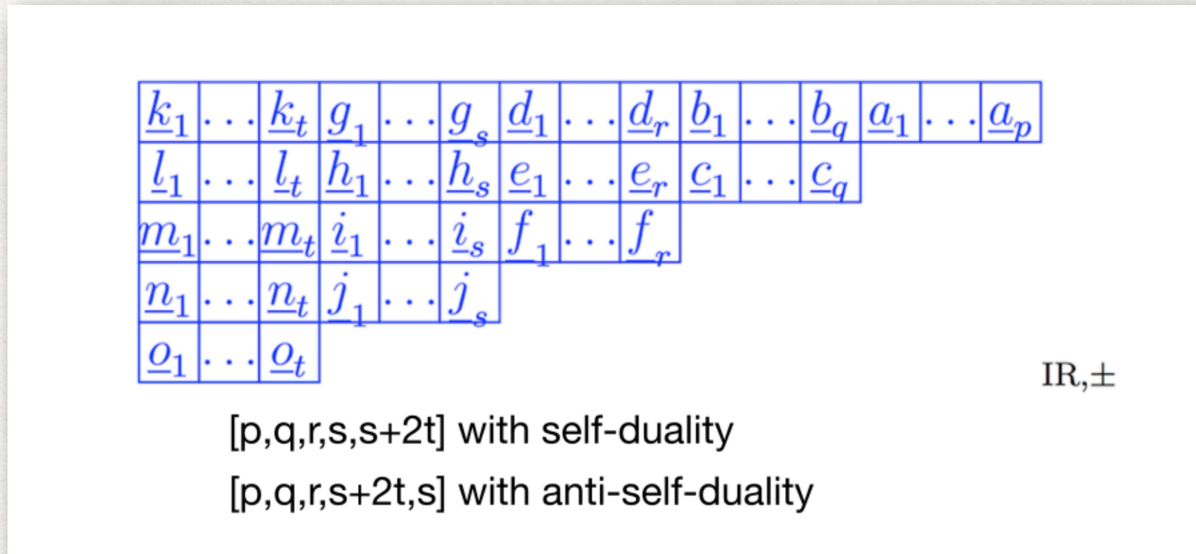
- Ordinary Young Tableaux in $SU(10)$: $[a, b, c, d, e, f, g, h, i]$

- Define the corresponding bosonic irrep in $SO(10)$ has the Dynkin label as $[a, b, c, d, d+2e]$:

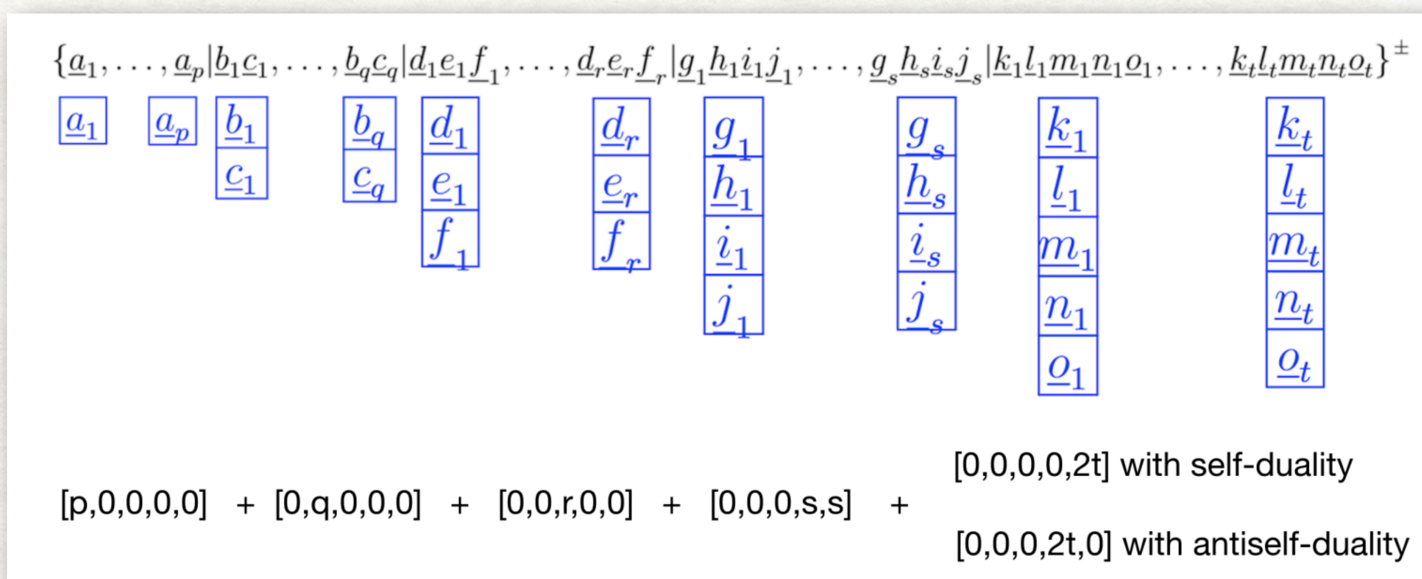
$$
[a, b, c, d, d+2e] = [a, b, c, d, e, 0, 0, 0, 0]\, P^T_{\mathfrak{su}(10)\supset\mathfrak{so}(10)}
$$

DICTIONARY: IRREP $\leftrightarrow$ FIELD VARIABLES

- Dynkin labels $\leftrightarrow$ BYT

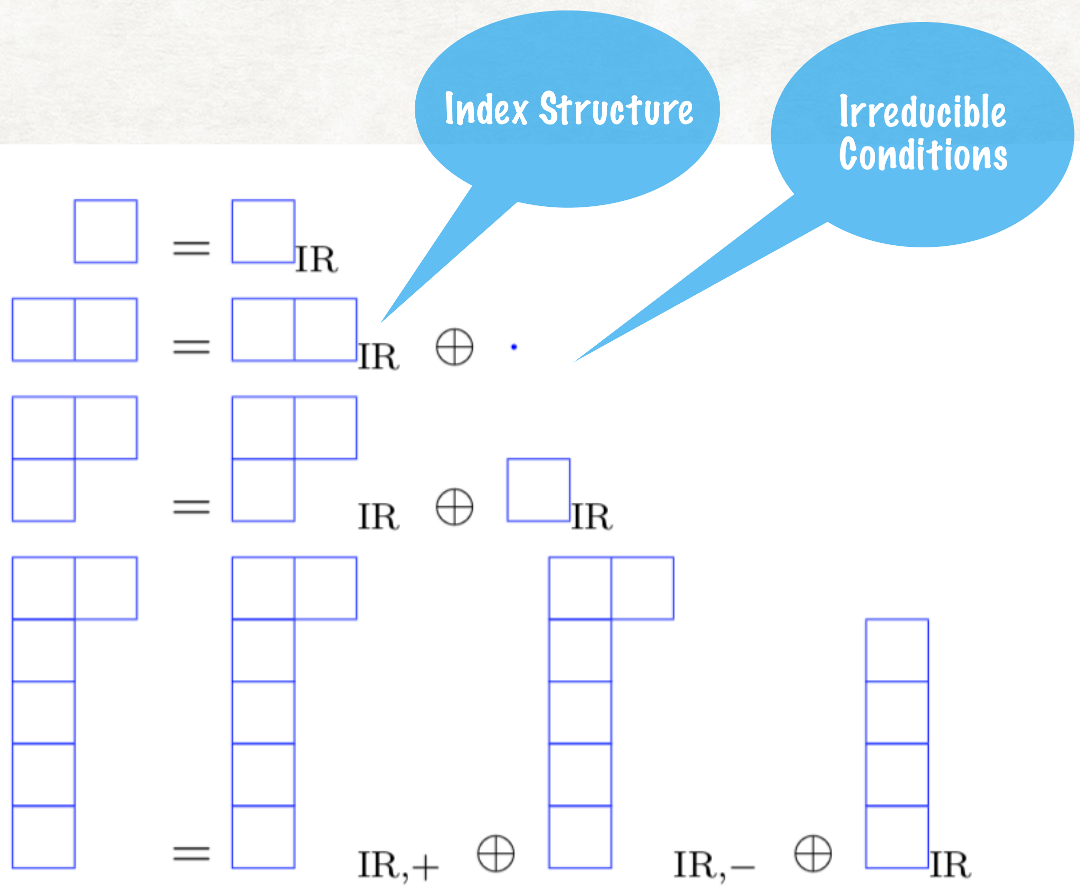


- BYT $\leftrightarrow$ Index structures of field variables



IRREDUCIBLE CONDITIONS

- Branching Rules for $\mathfrak{su}(10) \supset \mathfrak{so}(10)$ tell us the irreducible conditions



$$
\begin{aligned}
\Phi_{\{\underline{a}_1\}} &: \mathrm{N/A} \\
\Phi_{\{\underline{a}_1\underline{a}_2\}} &: \eta^{\underline{a}_1\underline{a}_2}\,\Phi_{\{\underline{a}_1\underline{a}_2\}} = 0 \\
\Phi_{\{\underline{a}_1\underline{a}_2|\underline{b}_1\}} &: \eta^{\underline{a}_1\underline{a}_2}\,\Phi_{\{\underline{a}_1\underline{a}_2|\underline{b}_1\}} = 0 \\
\Phi_{\{\underline{a}_1\underline{a}_2|\underline{b}_1|\underline{c}_1|\underline{d}_1|\underline{e}_1\}^+} &: \eta^{\underline{a}_1\underline{a}_2}\,\Phi_{\{\underline{a}_1\underline{a}_2|\underline{b}_1|\underline{c}_1|\underline{d}_1|\underline{e}_1\}^+} = 0 \\
&\quad \mathrm{self-dual\ condition} \\
\Phi_{\{\underline{a}_1\underline{a}_2|\underline{b}_1|\underline{c}_1|\underline{d}_1|\underline{e}_1\}^-} &: \eta^{\underline{a}_1\underline{a}_2}\,\Phi_{\{\underline{a}_1\underline{a}_2|\underline{b}_1|\underline{c}_1|\underline{d}_1|\underline{e}_1\}^-} = 0 \\
&\quad \mathrm{anti-self-dual\ condition}
\end{aligned}
$$

- D dimension: $\mathfrak{su}(\mathrm{D}) \supset \mathfrak{so}(\mathrm{D})$

10D IRREDUCIBLE SPINORIAL YOUNG TABLEAUX

- Two spinor indices → sigma matrix → vector indices
- Irreducible SYT ← Irreducible BYT $\otimes$ $\{16\}$ (or $\{\overline{16}\}$)

$$\{10\} \otimes \{16\} = \Box_{\rm IR} \otimes \{16\} = \{\overline{16}\} \oplus \{\overline{144}\}$$

$$\{45\} \otimes \{16\} = \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}_{\rm IR} \otimes \{16\} = \{16\} \oplus \{144\} \oplus \{560\}$$

Index Structure

Irreducible Conditions

$$\{16\} = \cdot \otimes \{16\} \qquad \Psi^{\alpha}$$

$$\{\overline{16}\} = \cdot \otimes \{\overline{16}\} \qquad \Psi_{\alpha}$$

$$\{560\} = \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}_{\rm IR} \otimes \{16\} - \Box_{\rm IR} \otimes \{\overline{16}\} \qquad \Psi_{\{\underline{a}|\underline{b}\}}{}^{\gamma} \ : \ (\sigma^{\underline{a}})_{\gamma\delta}\Psi_{\{\underline{a}|\underline{b}\}}{}^{\gamma} = 0$$

$$\{\overline{560}\} = \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}_{\rm IR} \otimes \{\overline{16}\} - \Box_{\rm IR} \otimes \{16\} \qquad \Psi_{\{\underline{a}|\underline{b}\}\gamma} \ : \ (\sigma^{\underline{a}})^{\gamma\delta}\Psi_{\{\underline{a}|\underline{b}\}\gamma} = 0$$

THE 4D, N=1MINIMAL SUPERMULTIPLET ZOO

(S01.) Chiral Supermultiplet : $(A,\, B,\, \psi_a,\, F,\, G)$,

(S02.) Hodge – Dual #1 Chiral Supermultiplet : $(\widehat{A},\, \widehat{B},\, \psi_a,\, \mathrm{f}_{\mu\,\nu\,\rho},\, \widehat{G})$,

(S03.) Hodge – Dual #2 Chiral Supermultiplet : $(\widetilde{A},\, \widetilde{B},\, \psi_a,\, \widehat{F},\, \mathrm{g}_{\mu\,\nu\,\rho})$,

(S04.) Hodge – Dual #3 Chiral Supermultiplet : $(\check{A},\, \check{B},\, \psi_a,\, \check{\mathrm{f}}_{\mu\,\nu\,\rho},\, \check{\mathrm{g}}_{\mu\,\nu\,\rho})$,

(S05.) Tensor Supermultiplet : $(\varphi,\, B_{\mu\,\nu},\, \chi_a)$,

(S06.) Axial – Tensor Supermultiplet : $(\widehat{\varphi},\, \widehat{B}_{\mu\,\nu},\, \widehat{\chi}_a)$,

(S07.) Vector Supermultiplet : $(A_\mu,\, \lambda_b,\, \mathrm{d})$,

(S08.) Axial – Vector Supermultiplet : $(U_\mu,\, \widehat{\lambda}_b,\, \widehat{\mathrm{d}})$,

(S09.) Hodge – Dual Vector Supermultiplet : $(\widetilde{A}_\mu,\, \widetilde{\lambda}_b,\, \widetilde{\mathrm{d}}_{\mu\,\nu\,\rho})$,

(S10.) Hodge – Dual Axial – Vector Supermultiplet : $(\check{U}_\mu,\, \check{\lambda}_b,\, \check{\mathrm{d}}_{\mu\,\nu\,\rho})$.

- Hodge duality relates some of the supermultiplets.
- Parity duality relates some of the supermultiplets.