Nonlinear $\mathcal{N}=2$ Supersymmetry and D2-brane Effective Actions

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1. Overview

Dp-branes acquire effective nonlinear descriptions whose bosonic parts are given by the Dirac-Born-Infeld action. This nonlinearity has been proven to be a consequence of the partial $\mathcal{N}=2\to\mathcal{N}=1$ supersymmetry breaking, originating from the solitonic nature of the branes. In this work, we focus on the effective descriptions of D2-branes, which play important roles in the Type IIA string theory.

- Using the Goldstone multiplet interpretation of the action and the method of nilpotent $\mathcal{N}=2$ superfields, we construct the 3D, $\mathcal{N}=1$ superspace effective action which makes the first supersymmetry manifest and realizes the second, spontaneously broken, supersymmetry nonlinearly.
- This description is in agreement with previously known results for the supersymmetric effective action of D2-branes [1, 2]. We show that there are two such effective superspace actions, which are generated by the 3D, $\mathcal{N}=2$ vector and tensor multiplets after expanding them around a nontrivial vacuum and enforcing constraints that eliminate additional degrees of freedom.
- We find that these descriptions are related by a duality transformation which results in the inversion of a dimensionless parameter. Moreover, we derive the explicit bosonic and fermionic parts of the effective spacetime actions.
- We consider the deformation of the Maxwell-Goldstone superspace action by the characteristic Chern-Simons-like, gauge-invariant, mass term of 3D vector multiplet.

2. Maxwell- & Tensor-Goldstone Multiplets

We aim to interpret the Maxwell and Tensor supermultiplets as the **Goldstone multiplets** that accommodates the Goldstinos associated with the **$\mathcal{N}=2\to\mathcal{N}=1$ spontaneous supersymmetry breaking** in 3D generated by the solitonic D2-brane solutions of type IIA string theory. The review of 3D, $\mathcal{N}=1$ Maxwell and Tensor supermultiplets is summarized in the following table.

	Field Strength	Bianchi Identity	Prepotential	Gauge Transformation
Maxwell	W_α	$\mathrm{D}^\alpha W_\alpha=0$	$W_\alpha=\frac{1}{2}\mathrm{D}^\beta\mathrm{D}_\alpha\Gamma_\beta$	$\delta\Gamma_\alpha=\mathrm{D}_\alpha K$
Tensor	$G\ (U_\alpha=\mathrm{D}_\alpha G)$	$\mathrm{D}^\alpha\mathrm{D}_\beta U_\alpha=0$	$G=\mathrm{D}^\alpha\tilde{\Gamma}_\alpha$	$\delta_G\tilde{\Gamma}_\alpha=\mathrm{D}^\beta\mathrm{D}_\alpha\Lambda_\beta,\ \delta_G\Lambda_\beta=\mathrm{D}_\beta\Lambda$

Bagger-Galperin Following [3], we search for the most general transformation δ^* of W_α (U_α) s.t.

- δ^* is consistent with Bianchi identities and can be interpreted as a SUSY transformation
- It must involve the second SUSY partner of W_α (U_α) which is a scalar $\mathcal{N}=1$ superfield X ($\tilde{X}$)
- It must contain a **characteristic shift** which is the hallmark of spontaneous SUSY breaking

To eliminate the independent degrees of freedom introduced by X ($\tilde{X}$) and promote the 3D Maxwell (Tensor) multiplet to a Maxwell(Tensor)-Goldstone multiplet, we impose **nonlinear constraints** which are consistent with δ^* and their solutions allow the construction of the superspace actions.

	Maxwell-Goldstone	Tensor-Goldstone
General transformation δ^*	$\delta^*_\eta W_\alpha=\eta_\alpha-\frac{1}{2\kappa}(\mathrm{D}^\beta\mathrm{D}_\alpha X)\,\eta_\beta$	$\delta^*_\eta U_\alpha=\eta_\alpha-\frac{1}{2\tilde{\kappa}}(\mathrm{D}_\alpha\mathrm{D}^\beta\tilde{X})\,\eta_\beta$
	$\delta^*_\eta X=\frac{2}{\kappa}\eta^\alpha W_\alpha$	$\delta^*_\eta\tilde{X}=\frac{2}{\tilde{\kappa}}\eta^\alpha U_\alpha$
Constraint	$\kappa X=W^\alpha W_\alpha+\frac{1}{2}(\mathrm{D}^2X)\,X$	$\tilde{\kappa}\tilde{X}=U^\alpha U_\alpha-\frac{1}{2}(\mathrm{D}^2\tilde{X})\,\tilde{X}$
Solution	$X=\frac{2}{\kappa}W^2\left[1+\frac{T}{1-T+\sqrt{1-2T}}\right]$	$\tilde{X}=\frac{2}{\tilde{\kappa}}U^2\left[1-\frac{\tilde{T}}{1+\tilde{T}+\sqrt{1+2\tilde{T}}}\right]$
Superspace Action	$S_{\mathsf{MG}}=\tau\int d^3x\,d^2\theta\,X$	$S_{\mathsf{TG}}=-\tilde{\tau}\int d^3x\,d^2\theta\,\tilde{X}$

Volkov-Akulov Following [4, 5], the above results can be easily understood from the viewpoint of the **manifest $\mathcal{N}=2$ theories** described by $\mathcal{W}(x,\theta,\tilde{\theta})$ and $\mathcal{U}(x,\theta,\tilde{\theta})$ respectively. To break the second manifest supersymmetry, we expand $\mathcal{W}$ and $\mathcal{U}$ around a **non-trivial vacuum** that preserves only the first supersymmetry and at the same time, we eliminate the remaining partner superfield by imposing the **nilpotency condition**. Then the second supersymmetry transformation laws reduce to δ^* and the nonlinear constraints match with what we obtained following Bagger and Galperin.

	$\mathcal{N}=2$ Vector Multiplet	$\mathcal{N}=2$ Tensor Multiplet
Scalar Superfield	$\mathcal{W}=\Phi(x,\theta)+\tilde{\theta}^\alpha W_\alpha(x,\theta)-\tilde{\theta}^2F(x,\theta)$	$\mathcal{U}=\Phi(x,\theta)+\tilde{\theta}^\alpha U_\alpha(x,\theta)-\tilde{\theta}^2F(x,\theta)$
Irreducibility Condition	$\mathrm{D}^2\mathcal{W}=\tilde{\mathrm{D}}^2\mathcal{W}$, $\mathrm{D}^\alpha\tilde{\mathrm{D}}_\alpha\mathcal{W}=0$	$\mathrm{D}^2\mathcal{U}=-\tilde{\mathrm{D}}^2\mathcal{U}$, $\mathrm{D}^\alpha\mathrm{D}_\beta\tilde{\mathrm{D}}_\alpha\mathcal{U}=0$
Nilpotency Condition	$\mathcal{W}=\langle\mathcal{W}\rangle+\mathbb{W}$, $\mathbb{W}^2=0$ $\mathrm{D}^\alpha\tilde{\mathrm{D}}_\alpha\mathcal{W}=0$	$\mathcal{U}=\langle\mathcal{U}\rangle+\mathbb{U}$, $\mathbb{U}^2=0$
Condensate	$\langle\mathcal{W}\rangle=\kappa\,\tilde{\theta}^2$	$\langle\mathcal{U}\rangle=\tilde{\kappa}\,\tilde{\theta}^2$

D2-brane Actions Starting from the superspace action, we extract the spacetime component action, which can be written as a member of the Cecotti-Ferrara class of actions

$$S_{\mathsf{CF}}=\tau\,\kappa\int d^3x\left\{T\big|_{\theta=0}+\frac{2}{\kappa^2}\int d^2\theta\,\Psi(T)\,W^2\right\}\ . \tag{1}$$

In our cases, $\Psi(x)=\frac{x}{1-x+\sqrt{1-2x}}$. The bosonic part of the effective superspace D2-brane action is the 3D **Born-Infeld** theory as expected.

	Maxwell-Goldstone	Tensor-Goldstone
Bosonic	$S_{\mathsf{B}}=\tau\kappa\int d^3x\Big(1-\sqrt{1-2s}\Big)$	$S_{\mathsf{B}}=\tilde{\tau}\tilde{\kappa}\int d^3x\Big(1-\sqrt{1+2\tilde{s}}\Big)$
Fermionic	$S_{\mathsf{F}}=\frac{\tau}{\kappa^3}\int d^3x\Big\{\Psi'(T\vert)\Big[4i(f^{\alpha\delta}\lambda_\delta)\partial_{\alpha\beta}(f^{\beta\gamma}\lambda_\gamma)$	$S_{\mathsf{F}}=\frac{\tilde{\tau}}{\tilde{\kappa}^3}\int d^3x\Big\{\Psi'(-\tilde{T}\vert)\Big[4i\partial_{\alpha\beta}(\hat{f}^{\beta\gamma}\chi_\gamma)\hat{f}^{\alpha\delta}\chi_\delta$
	$+\Big(2(\Box\lambda^\gamma)\lambda_\gamma+(\partial^{\alpha\beta}\lambda^\gamma)(\partial_{\alpha\beta}\lambda_\gamma)\Big)\lambda^\sigma\lambda_\sigma\Big]$	$+\Big(2(\Box\chi^\gamma)\chi_\gamma+(\partial^{\alpha\beta}\chi^\gamma)(\partial_{\alpha\beta}\chi_\gamma)\Big)\chi^\sigma\chi_\sigma\Big]$
	$-\frac{2}{\kappa^2}\Psi''(T\vert)\Big[\partial^\alpha{}_\beta(f^{\beta\gamma}\lambda_\gamma)\Big]\Big[\partial_{\alpha\delta}(f^{\delta\epsilon}\lambda_\epsilon)\Big]\lambda^\sigma\lambda_\sigma\Big\}$	$+\frac{2}{\tilde{\kappa}^2}\Psi''(-\tilde{T}\vert)\Big[\partial^\alpha{}_\beta(f^{\beta\gamma}\lambda_\gamma)\Big]\Big[\partial_{\alpha\delta}(f^{\delta\epsilon}\lambda_\epsilon)\Big]\chi^\sigma\chi_\sigma\Big\}$

where $T| = \frac{2}{\kappa^2}\Big[-\frac{1}{2}f^{\alpha\beta}f_{\alpha\beta}-i\lambda^\alpha(\partial_{\alpha\beta}\lambda^\beta)\Big]$ and $s=-\frac{1}{\kappa^2}f^{\alpha\beta}f_{\alpha\beta}$;
$\tilde{T}|=\frac{2}{\tilde{\kappa}^2}\Big[-\frac{1}{2}\hat{f}^{\alpha\beta}\hat{f}_{\alpha\beta}+i\chi^\alpha(\partial_{\alpha\beta}\chi^\beta)\Big]$ and $\tilde{s}=-\frac{1}{\tilde{\kappa}^2}\hat{f}^{\alpha\beta}\hat{f}_{\alpha\beta}$.

3. Duality

In 3D, the vector multiplet is no longer self-dual as in 4D but it maps to the tensor multiplet. We show that this duality map survives between the Maxwell-Goldstone and Tensor-Goldstone multiplets. We start with the following "parent action" and show that it can generate Maxwell- and Tensor-Goldstone multiplets respectively by integrating out superfields in different orders.

In this description, superfields Λ, X, and W_α are unconstrained and the duality term **Lagrange multiplier U_α** satisfies the constraint $\mathrm{D}^\alpha\mathrm{D}_\beta U_\alpha=0$ (Tensor multiplet).

$$S_{\mathsf{P.A.}}=\int d^3x\,d^2\theta\left\{\Lambda\left[W^\alpha W_\alpha+\frac{1}{2}X\,\mathrm{D}^2X-\kappa X\right]+\tau X+g\,\mathbf{U}^\alpha\mathbf{W}_\alpha\right\}$$

Integrating out $\mathbf{U}_\alpha$ first restores the vector constraint $\mathrm{D}^\alpha W_\alpha=0$; the **Lagrange multiplier** Λ imposes the nonlinear constraint for **X**. The resulting action is $\mathbf{S}_{\mathsf{MG}}$.

Integrate out $\mathbf{W}_\alpha$ first, the result is the following
$$S=\int d^3x\,d^2\theta\left\{\tilde{\Lambda}\left[U^\alpha U_\alpha-\frac{1}{2}\tilde{X}\mathrm{D}^2\tilde{X}-\tilde{\kappa}\tilde{X}\right]-\tilde{\tau}\tilde{X}\right\}$$
where $\tilde{\Lambda}=-\frac{g^2}{4\Lambda}$, $X=\frac{g\tilde{X}}{2\Lambda}$, $\tilde{\kappa}=\frac{2\tau}{g}$, $\tilde{\tau}=\frac{g\kappa}{2}$.
This action corresponds to the $\mathbf{S}_{\mathsf{TG}}$, since the **Lagrange multiplier** $\tilde{\Lambda}$ imposes the nonlinear constraint for $\tilde{\mathbf{X}}$.

The **dimensionless** parameter $\lambda=\frac{\kappa}{\tau}$ undergoes a standard **inversion** $\lambda\to\tilde{\lambda}=\frac{4}{g^2\lambda}$.

4. Chern-Simons-type Mass Deformation

A special property of the 3D vector multiplet which makes it very different from its 4D analog is the existence of a gauge-invariant Chern-Simons mass term $S_m=\frac{m}{2}\int d^3x\,d^2\theta\,\Gamma^\alpha W_\alpha$. After examining the gauge and the second supersymmetry transformation, we find that there exists a **one-parameter (ξ) family of deformations** of the Maxwell-Goldstone action

$$S_\xi=\frac{\xi\,m}{2}\int d^3x\,d^2\theta\left\{\Gamma^\alpha W_\alpha-2\kappa\,\theta^\alpha\Delta_\alpha\right\}\ . \tag{2}$$

- S_ξ respects the second supersymmetry transformations (3) and gauge transformations.
- The superfield Δ_α is defined as $\mathrm{D}^\alpha\Delta_\alpha=X$, which enjoys the gauge transformation $\delta\Delta_\alpha=\mathrm{D}^\beta\mathrm{D}_\alpha K_\beta$ and $\delta K_\alpha=\mathrm{D}_\alpha K$.
- The second supersymmetry transformations of Γ_α and Δ_α are
$$\delta^*\Gamma_\alpha=\frac{1}{\kappa}\eta^\beta\mathrm{D}_\beta\Delta_\alpha+\Phi^{(sp)}_\alpha\ ,\ \delta^*\Delta_\alpha=-\frac{1}{\kappa}\eta^\beta\mathrm{D}_\beta\Gamma_\alpha\ . \tag{3}$$
$\Phi^{(sp)}_\alpha=-2\eta_\alpha\theta^2$ is the special solution of the inhomogeneous equation $\mathrm{D}^\beta\mathrm{D}_\alpha\Phi^{(sp)}_\beta=2\eta_\alpha$. The full solution of the homogeneous equation $\mathrm{D}^\beta\mathrm{D}_\alpha\Phi_\beta=0$ corresponds to a gauge transformation of Γ_α and thus can be dropped.
- The second term explicitly breaks the first supersymmetry.

References

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